

# Metric Classification of Geometries of Positive Ricci Curvature in 3D

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Using Cartan's approach, [C1946], we give an explicit formula of the metric tensor of a simply connected 3D homogeneous space of positive scalar curvature in terms of the principal Ricci curvatures at one point. The principal Ricci curvatures at a single point fully determine such an isometry class of a 3D Riemannian homogeneous space, and each isometry class can be represented by Lie group with a left invariant metric. The problem raised in [P2002] that a 3D compact connected simply connected manifold has a Lie group structure is an open alternative to the Ricci flow approach [Pe2002] [Pe2003] to Poincaré's conjecture.

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