Future Directions for Research in Geometry: A Summary of the Special Discussion Session at the 2005 Midwest Geometry Conference

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The 2005 Midwest Geometry Conference took place at the Ohio State University from April 29 to May 1, 2005. The conference included a special session on the future directions of geometry. During this session, the conference participants discussed some of the areas within the field of geometry in which significant research activity is likely to occur over the next few years. These notes summarize the discussion during the special session.

One of the first topics that came up at the future directions session was the connection between geometry on the one hand and particle physics, gauge theory, Kaluza-Klein theory, and string theory on other. As in the past, these connections are expected to generate a significant amount of research activity in geometry in the coming years.

Participants at the special discussion session mentioned other areas of geometry as well as some of their applications and their connections with other fields within science and mathematics. Research in geometry will in all likelihood continue to have connections with research in ordinary and partial differential equations. Participants mentioned Poisson geometry and noted that this field is expected to have applications to plasma systems. In the coming years, as in the past, connections between geometry, knot theory, and the study of DNA molecules are expected to stimulate research in geometry. Morse theory has been applied in the study of computer imaging, and participants at the discussion session felt that these applications were likely to continue.

Some participants at the discussion session indicated that they study geometry for its own sake, without regard for its applications. And yet geometry does indeed have many practical applications. Participants noted that applications of geometric theory may occur many years after the research that originally produced it. To find examples of such geometric theory, one need only consider the field of Riemannian geometry. Riemannian geometry served as a tool in the development of Einstein's general theory of relativity, over fifty years after the discovery of Riemannian geometry itself. It is also worth noting that certain applications of Albert Einstein's work occurred many years after the work itself. For example, the construction of nuclear power plants and atomic bombs depended in part on Einstein's idea of mass-energy equivalence, an idea that Einstein formulated over thirty years prior to the applications themselves. These examples illustrate the long-term value of continued basic research in geometry and related fields.

Participants at the special session on future directions of geometry also discussed graduate students and the question of suitable problems for them to work on. The following geometry-related areas contain many problems suitable for graduate students: symplectic geometry, Poisson geometry, analysis on manifolds, heat asymptotics, quantum gravity, conformal geometry, noncommutative geometry, and knot theory. Success in working on many of the open problems in these fields will require the student to have an extensive amount of background knowledge in a wide variety of mathematical areas. For this reason, geometry will continue to attract talented people who seek challenging opportunities. On the other hand, participants also pointed out that there are also open problems in geometry which are suitable for graduate students with a more moderate level of experience and background knowledge; one participant noted that certain problems in noncommutative geometry are of this nature.

To summarize the discussion at the special session on future directions, it suffices to say that geometry is a broad field with many opportunities for research and many connections with other disciplines. Geometry will likely continue to attract many graduate students and to provide a wide range of benefits to society over the long term.

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