We construct the surface $\mathcal{R}$, consisting in homotopy classes[7] of smooth curves in $\mathcal{W}$ starting at the origin, moving away from it, and crossing at most one singular line, at most once (see Fig. 5.1):

$$\mathcal{R} := \left\{\gamma : (0,1) \mapsto \mathcal{W} : \ \gamma(0_+) = 0; \ \frac{\mathrm{d}}{\mathrm{d}t}|\gamma(t)| > 0; \ \frac{\mathrm{d}}{\mathrm{d}t}\arg(\gamma(t)) \text{ monotonic}\right\}$$
$$\text{(modulo homotopies)} \tag{5.66}$$

Define $\mathcal{R}_1 \subset \mathcal{R}$ by (5.66) with the supplementary restriction $\arg\gamma \in (\max\{\arg\lambda_j : \arg\lambda_j < 0\}, \min\{\arg\lambda_j : \arg\lambda_j > 0\})$, now with arg in $(-\pi, \pi)$.

$\mathcal{R}_1$ may be viewed as the part of $\mathcal{R}$ constructed over a sector containing $\mathbb{R}^+$.

(Similarly we let $\mathcal{R}_j \subset \mathcal{R}$ with the restriction that the curves γ do not cross any singular direction *other* than $e^{i\phi_j}\mathbb{R}^+$.) We let $\psi_\pm = \pm\max(\pm\arg\gamma)$ with $\gamma \in \mathcal{R}_1$.

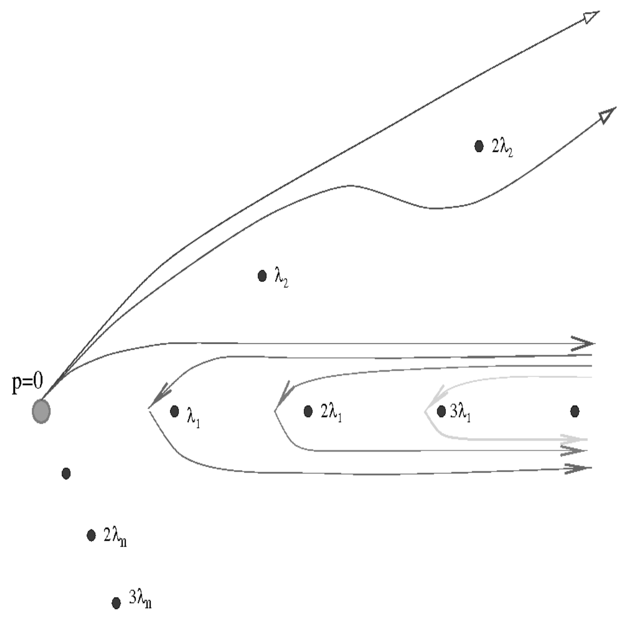

FIGURE 5.1: The paths near λ_1 relate to the medianized averages.

By symmetry (renumbering the directions) it suffices to analyze the singu-

[7] Classes of curves that can be continuously deformed into each other without crossing points outside $\mathcal{W}$.