5.10j Equations and properties of $\mathbf{Y_k}$ and summation of the transseries

Proposition 5.215 *Let* $\mathbf{Y}$ *be any* $L^1_{\rm loc}(\mathbb{R}^+)$ *solution of (5.89). For large* ν *and some* $c > 0$ *the coefficients* $\mathbf{D_j}$ *in (5.92) are bounded by*

$$|\mathbf{D_j}(p)| \le e^{c_0 p} c^{|\mathbf{m}|}$$

Note that $\mathcal{L}^{-1}(\mathbf{g}^{(\mathbf{m})}(x,\mathbf{y})/\mathbf{m}!)$ is the coefficient of $\mathbf{Z}^{*\mathbf{m}}$ in the expansion of $\mathcal{N}(\mathbf{Y}+\mathbf{Z})$ in convolution powers of Z (5.87):

$$\left(\left(\sum_{|\mathbf{l}|\ge \mathbf{2}} \mathbf{g}_{0,\mathbf{l}}\cdot + \sum_{|\mathbf{l}|\ge \mathbf{1}} \mathbf{G_l}*\right)(\mathbf{Y}+\mathbf{Z})^{*\mathbf{l}}\right)_{\mathbf{Z}^{*\mathbf{m}}} =$$
$$\left(\left(\sum_{|\mathbf{l}|\ge \mathbf{2}} \mathbf{g}_{0,\mathbf{l}}\cdot + \sum_{|\mathbf{l}|\ge \mathbf{1}} \mathbf{G_l}*\right)\sum_{0\le \mathbf{k}\le \mathbf{l}} \binom{\mathbf{l}}{\mathbf{k}} \mathbf{Z}^{*\mathbf{k}}\mathbf{Y}^{*(\mathbf{l}-\mathbf{k})}\right)_{\mathbf{Z}^{*\mathbf{m}}} =$$
$$\left(\sum_{|\mathbf{l}|\ge \mathbf{2}} \mathbf{g}_{0,\mathbf{l}}\cdot + \sum_{|\mathbf{l}|\ge \mathbf{1}} \mathbf{G_l}*\right)\sum_{\mathbf{l}\ge\mathbf{m}} \binom{\mathbf{l}}{\mathbf{m}} \mathbf{Y}^{*(\mathbf{l}-\mathbf{m})} \tag{5.216}$$

($\mathbf{m}$ is fixed) where $\mathbf{l}\ge\mathbf{m}$ means $l_i \ge m_i, i = 1,...,n$ and $\binom{\mathbf{l}}{\mathbf{k}} := \prod_{i=1}^n \binom{l_i}{k_i}$.

Let ϵ be small and ν large so that $\|\mathbf{Y}\|_\nu < \epsilon$. Then, for some constant K, we have (cf. (5.135))

$$\left\|\left(\sum_{II} \mathbf{g}_{0,\mathbf{l}}\cdot + \sum_{I} \mathbf{G_l}*\right)\binom{\mathbf{l}}{\mathbf{m}} \mathbf{G_l} * \mathbf{Y}^{*(\mathbf{l}-\mathbf{m})}\right\|_\nu \le \sum_I c_0^{-|\mathbf{l}|} K e^{c_0|p|}(c_0\epsilon)^{|\mathbf{l}-\mathbf{m}|}\binom{\mathbf{l}}{\mathbf{m}} =$$

$$\epsilon^{-|\mathbf{m}|} K e^{c_0|p|}\prod_{i=1}^n \sum_{l_i\ge m_i}\binom{l_i}{m_i}(c_0\epsilon)^{l_i} = K\frac{e^{c_0|p|}{c_0}^{|\mathbf{m}|}}{(1-\epsilon c_0)^{|\mathbf{m}|+n}} \le e^{c_0|p|}c^{|\mathbf{m}|} \tag{5.217}$$

(where $I(II, resp.) \equiv \{|\mathbf{l}| \ge 1(2, resp.); \mathbf{l}\ge\mathbf{m}\}$) for large enough ν.

□

For $k = 1$, $\mathbf{T}_1 = 0$ and equation (5.92) is (5.191) (with $p \leftrightarrow z$) but now on the whole line $\mathbb{R}^+$. For small $|z|$ the solution is given by (5.196) (note that $\mathbf{D}_1 = \mathbf{d}_{(1,0,\dots,0)}$ and so on) and depends on the free constant C (5.196). We choose a value for C (the values of $\mathbf{Y}_1$ on $[0,\epsilon]$ are then determined) and we write the equation of $\mathbf{Y}_1$ for $p \ge \epsilon$ as