6.6 Further examples

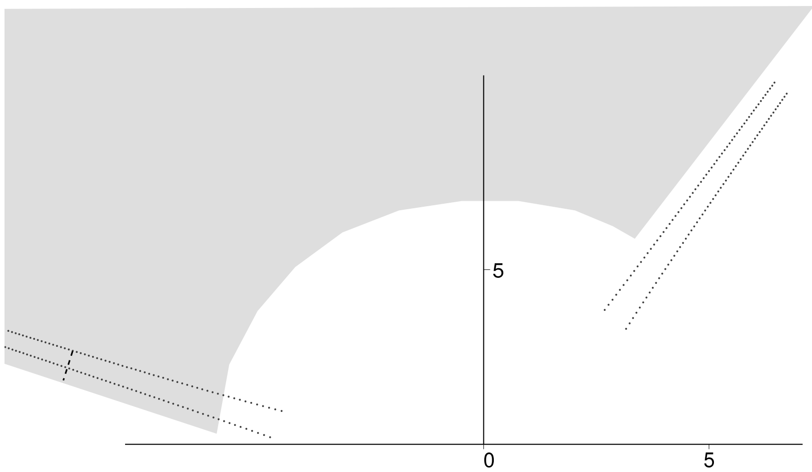

FIGURE 6.3: Singularities on the boundary of S_t for (5.52). The gray region lies in the projection on $\mathbb{C}$ of the Riemann surface where (6.29) holds. The short dotted line is a generic cut delimiting a first Riemann sheet.

6.6a The Painlevé equation $\mathrm{P_I}$

Proposition 6.62 below shows, in (i), how the constant C beyond all orders is associated to a truncated solution $y(z)$ of $\mathrm{P_I}$ for $\arg(z) = \pi$ (formula (6.63)) and gives the position of one array of poles z_n of the solution associated to C (formula (6.64)), and in (ii) provides uniform asymptotic expansion to all orders of this solution in a sector centered on $\arg(z) = \pi$ and one array of poles (except for small neighborhoods of these poles) in formula (6.66). Here, the rearranged transseries is $\tilde{y} = -\sqrt{-z/6}\sum_{k=0}^{\infty} \xi^k \tilde{y}_k(z^{-5/2})$, *cf.* [24]; the normalized variable is $x = x(z) = (-24z)^{5/4}/30$ and now $\xi = x^{-1/2}e^{-x}$.

Proposition 6.62 *(i) Let y be a solution of (4.81) such that $y(z) \sim \sqrt{-z/6}$ for large z with* $\arg(z) = \pi$*. For any $\phi \in (\pi, \pi + \frac{2}{5}\pi)$ the following limit*