where $\mathbf{u} \in \mathbb{C}^m$, for large $|\mathbf{x}|$ in S. Generically, the constant coefficient operator $\mathcal{P}(\partial_{\mathbf{x}})$ in the linearization of $\mathbf{g}(\infty, t, \cdot)$ is diagonalizable. It is then taken to be diagonal, with eigenvalues $\mathcal{P}_j$. $\mathcal{P}$ is subject to the requirement that for all $j \le m$ and $\mathbf{p} \ne 0$ in $\mathbb{C}^d$ with $|\arg p_i| \le \phi$ we have

$$\operatorname{Re} \mathcal{P}_j^{[n]}(-\mathbf{p}) > 0 \tag{7.21}$$

where $\mathcal{P}^{[n]}(\partial_{\mathbf{x}})$ is the principal symbol of $\mathcal{P}(\partial_{\mathbf{x}})$. Then the following holds. (The precise conditions and results are given in [21].)

Theorem 7.22 (large $|\mathbf{x}|$ existence) *Under the assumptions above, for any $T > 0$ (7.20) has a unique solution $\mathbf{u}$ that for $t \in [0, T]$ is $O(|\mathbf{x}|^{-1})$ and analytic in $\mathcal{S}$.*

Determining asymptotic properties of solutions of PDEs is substantially more difficult than the corresponding question for ODEs. Borel-Laplace techniques, however, provide a very efficient way to overcome this difficulty. The paper shows that formal series solutions are actually Borel summable, a fortiori asymptotic, to actual solutions. The restrictions on $\mathbf{g}_1$, $\mathbf{g}_2$, and $\mathbf{u_I}$ are simpler in a normalized form, obtained by simple transformations cf. [21], more suitable for our analysis,

$$\partial_t \mathbf{f} + \mathcal{P}(\partial_{\mathbf{x}})\mathbf{f} = {\sum_{\mathbf{q} \succeq 0}}' \mathbf{b_q}(\mathbf{x}, t, \mathbf{f}) \prod_{l, |\mathbf{j}|} \left(\partial_{\mathbf{x}}^{\mathbf{j}} f_l\right)^{q_{l,\mathbf{j}}} + \mathbf{r}(\mathbf{x}, t) \quad \text{with} \quad \mathbf{f}(\mathbf{x}, 0) = \mathbf{f}_I(\mathbf{x}) \tag{7.23}$$

where $\sum'$ means the sum over the multiindices $\mathbf{q}$ with

$$\sum_{l=1}^{m} \sum_{1 \le |\mathbf{j}| \le n} |\mathbf{j}| q_{l,\mathbf{j}} \le n \tag{7.24}$$

Condition 7.25 *The functions $\mathbf{b_{q,k}}(\mathbf{x}, t)$ and $\mathbf{r}(\mathbf{x}, t)$ are analytic in $(x_1^{-\frac{1}{N_1}}, ..., x_d^{-\frac{1}{N_d}})$ for large $|\mathbf{x}|$ and some $\mathbf{N} \in (\mathbb{N}^+)^d$.*

Theorem 7.26 *If Condition 7.25 and the assumptions of Theorem 7.22 are satisfied, then the unique solution $\mathbf{f}$ found there can be written as*

$$\mathbf{f}(\mathbf{x}, t) = \int_{\mathbb{R}^{+d}} e^{-\mathbf{p} \cdot \mathbf{x}^{\frac{n}{n-1}}} \mathbf{F}_{1^+}(\mathbf{p}, t) d\mathbf{p} \tag{7.27}$$

where $\mathbf{F}_{1^+}$ is (i) analytic at zero in $(p_1^{\frac{1}{nN_1}}, ..., p_d^{\frac{1}{nN_d}})$; (ii) analytic in $\mathbf{p} \ne \mathbf{0}$ in the poly-sector $|\arg p_i| < \frac{n}{n-1}\phi + \frac{\pi}{2(n-1)}$, $i \le d$; and (iii) exponentially bounded in the latter poly-sector.

Existence and asymptoticity of the formal power series follow as a corollary, using Watson's lemma.

The analysis has been extended recently to the Navier-Stokes system in $\mathbb{R}^3$; see [26].