the only consistent balance[8] is between $-\epsilon^2 w'^2$ and $V(x)-E$ with $\epsilon^2 w''$ much smaller than both. For that to happen we need

$$\epsilon^2 U^{-1} h' \ll 1 \quad \text{where} \quad h = w' \tag{3.193}$$

We place the term $\epsilon^2 h'$ on the right side of the equation and set up the iteration scheme

$$h_n^2 = \epsilon^{-2} U - h'_{n-1}; \quad h_{-1} = 0 \tag{3.194}$$

or

$$h_n = \pm \frac{\sqrt{U}}{\epsilon} \sqrt{1 - \frac{\epsilon^2 h'_{n-1}}{U}}; \quad h_{-1} = 0 \tag{3.195}$$

Under the condition (3.193) the square root can be Taylor expanded around 1,

$$h_n = \pm \frac{\sqrt{U}}{\epsilon} \left(1 - \frac{1}{2}\epsilon^2 \frac{h'_{n-1}}{U} - \frac{1}{8}\epsilon^4 \left(\frac{h'_{n-1}}{U} \right)^2 + \cdots \right) \tag{3.196}$$

We thus have

$$h_0 = \pm \epsilon^{-1} U^{1/2} \tag{3.197}$$

$$h_1 = \pm \epsilon^{-1} U^{1/2} \left(1 \mp \frac{1}{2}\epsilon^2 \frac{h'_0}{U} \right) = \pm \epsilon^{-1} U^{1/2} - \frac{1}{4}\frac{U'}{U} \tag{3.198}$$

$$h_2 = \pm \epsilon^{-1} U^{1/2} - \frac{1}{4}\frac{U'}{U} \pm \epsilon \left(-\frac{5}{32}\frac{(U')^2}{U^{5/2}} + \frac{1}{8}\frac{U''}{U^{3/2}} \right) \tag{3.199}$$

and so on. We can check that the procedure is formally sound if $\epsilon^2 U^{-1} h'_0 \ll 1$ or

$$\epsilon U' U^{-3/2} \ll 1 \tag{3.200}$$

Formally we have

$$w = \pm \epsilon^{-1} \int U^{1/2}(s) ds - \frac{1}{4} \ln U + \cdots \tag{3.201}$$

and thus

$$\psi \sim U^{-1/4} e^{\pm \epsilon^{-1} \int U^{1/2}(s) ds} \tag{3.202}$$

[8]As the parameter, ϵ in our case, gets small, various terms in the equation contribute unevenly. Some become relatively large (the dominant ones) and some are small (the subdominant ones). If no better approach is presented, one tries all possible combinations, and rules out those which lead to conclusions inconsistent with the size assumptions made. The approach roughly described here is known as the method of dominant balance [6]. It is efficient but heuristic and has to be supplemented by rigorous proofs at a later stage of the analysis.