

# CORRECTIONS TO “EXPANSIONS OF O-MINIMAL STRUCTURES BY DENSE INDEPENDENT SETS” (AND SOME OTHER RELATED PAPERS)

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There is a gap in the proof of [4, 2.19]: It is only clear that  $F$  is given piecewise off an  $H$ -small set by functions definable in  $\mathfrak{M}$ . This has already been repaired elsewhere, as we now explain.

The gap was discovered by E. Baro, in joint work with A. Martin-Pizarro, in the setting of dense pairs as defined in [5]. They could see in the proof of [5, 3.4] how “familiar model theoretic arguments” yield that the function  $F$  is given piecewise off a small set by finitely many functions definable in  $\mathcal{B}$ , but it was not obvious as to why  $F$  is given piecewise off a small set by *one* function definable in  $\mathcal{B}$ . They constructed a proof, but it was more involved than the text of [5] seemed to suggest. Baro then contacted the author of [5], L. van den Dries, who conceded that passing from finitely many functions to just one appears to be more complicated than he must have thought. Van den Dries was aware that some other authors had relied on [5, 3.4] or its proof (e.g., [2, 3, 6]); he contacted some of them, including Miller.

The gap in the proof of [5, 3.4] was repaired by Baro and Martin-Pizarro in a more abstract setting (see [1, Appendix]), thus confirming results in [2, 3, 6] that relied on [5, 3.4] or its proof. But as we had also thought that [4, 2.19] had the same proof as [5, 3.4], the gap would need to be repaired there as well. Happily, it appears to us that the fix in [1, Appendix] for dense pairs also works for the setting of [4].

A minor typesetting oversight: In the statement of [4, 2.20], “4.3” should be “[13, 4.3]”.

## REFERENCES

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