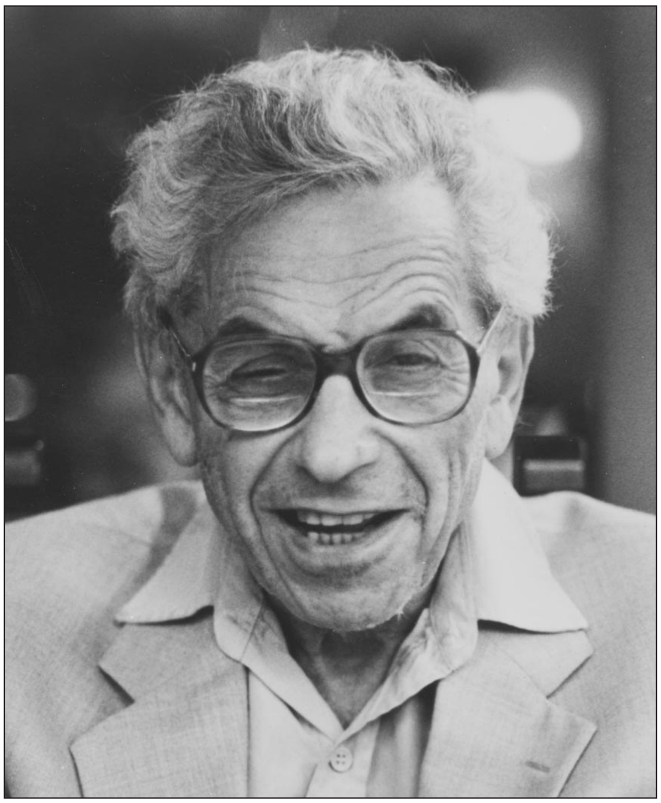
Paul Erdős (1913–1996)

László Babai and Joel Spencer



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Uncle Paul

Joel Spencer

Paul Erdős was a searcher, a searcher for mathematical truth.

Paul's place in the mathematical pantheon will be a matter of strong debate, for in that rarefied atmosphere he had a unique style. The late Ernst Straus said it best in a commemoration of Erdős's seventieth birthday.

> In our century, in which mathematics is so strongly dominated by "theory constructors" he has remained the prince of problem solvers and the absolute monarch of problem posers. One of my friends—a great mathematician in his own right—complained to me that "Erdős only gives us corollaries of the great metatheorems which remain unformulated in the back of his mind." I think there is much truth to that observation, but I don't agree that it would have been either feasible or desirable for Erdős to stop producing corollaries and concentrate on the formulation of his metatheorems. In many ways Paul Erdős is the Euler of our times. Just as the "special" problems that Euler solved pointed the way to analytic and algebraic number theory, topology, combinatorics, function spaces, etc., so the methods and results of Erdős's work already let us see the outline of great new disciplines, such as combinatorial and probabilistic number theory, combinatorial geometry, probabilistic and transfinite combinatorics and graph theory, as well as many more yet to arise from his ideas.

Straus, who worked as an assistant to Albert Einstein, noted that Einstein chose physics over mathematics because he feared that one would waste one's powers in pursuing the many beautiful and attractive questions of mathematics without finding the central questions. Straus goes on,

> Erdős has consistently and successfully violated every one of Einstein's prescriptions. He has succumbed to the seduction of every beautiful problem he

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This is an adaptation of a talk given in "A Tribute to Paul Erdős" at the San Diego meeting of the AMS in January 1997.

> has encountered—and a great many have succumbed to him. This just proves to me that in the search for truth there is room for Don Juan's like Erdős and Sir Galahad's like Einstein.

I believe, and I am certainly most prejudiced on this score, that Paul's legacy will be strongest in discrete math. Paul's interest in this area dates back to a marvelous paper with George Szekeres in 1935, but it was after World War II that it really flourished. The rise of the discrete over the past half century has, I feel, two main causes. The first was the computer—how wonderful that this physical object has led to such intriguing mathematical questions. The second, with due respect to the many others, was the constant attention of Paul Erdős, with his famous admonition "Prove and conjecture!" Ramsey theory, extremal graph theory, random graphs—how many turrets in our mathematical castle were built one brick at a time with Paul's theorems and, equally important, his frequent and always penetrating conjectures.

My own research specialty, the probabilistic method, could surely be called the Erdős method. It was begun in 1947 with a three-page paper in the *Bulletin of the American Mathematical Society*. Paul proved the existence of a graph having certain Ramsey property without actually constructing it. In modern language he showed that an appropriately defined random graph would have the property with positive probability and hence that there must exist a graph with the property. For the next twenty years Paul was a "voice in the wilderness"; his colleagues admired his amazing results, but adaption of the methodology was slow. But Paul persevered—he was always driven by his personal sense of mathematical aesthetics, in which he had supreme confidence—and today the method is widely used in both discrete math and theoretical computer science.

There is no dispute over Paul's contribution to the spirit of mathematics. Paul Erdős was the most inspirational man I have ever met. I began working with Paul in the late 1960s, a tumultuous time when "do your own thing" was the admonition that resonated so powerfully. But while others spoke of it, this was Paul's modus operandi. He had no job; he worked constantly. He had no home; the world was his home. Possessions were a nuisance, money a bore. He lived on a web of trust, travelling ceaselessly from center to center, spreading his mathematical pollen.

What drew so many of us into his circle? What explains the joy we have in speaking of this gentle man? Why do we love to tell Erdős stories? I have thought a great deal about this, and I think it comes down to a matter of belief or faith. We mathematicians know the beauties of our subject, and we hold a belief in its transcendent quality. God created the integers, the rest is the work of Man. Mathematical truth is immutable; it lies outside physical reality. When we show, for example, that two n-th powers never add up to an n-th power for $n \geq 3$, we have discovered a truth. This is our belief; this is our core motivating force. Yet our attempts to describe this belief to our nonmathematical friends are akin to describing the Almighty to an atheist. Paul embodied this belief in mathematical truth. His enormous talents and energies were given entirely to the Temple of Mathematics. He harbored no doubts about the importance, the absoluteness, of his quest. To see his faith was to be given faith. The religious world might better have understood Paul's special personal qualities. We knew him as Uncle Paul.

I do hope that one cornerstone of Paul's theology, if you will, will long survive. I refer to *The Book*. *The Book* consists of all the theorems of mathematics. For each theorem there is in *The Book* just one proof. It is the most aesthetic proof, the most insightful proof, what Paul called *The Book* proof. When one of Paul's myriad conjectures was resolved in an "ugly" way, Paul would be very happy to congratulate the prover, but would add, "Now, let's look for *The Book* proof." This platonic ideal spoke strongly to those of us in his circle. The mathematics was there; we had only to discover it. The intensity and the selflessness of the search for truth were described by the writer Jorge Luis Borges in his story "The Library of Babel". The narrator is a worker in this library, which contains on its infinite shelves all wisdom. He wanders its infinite corridors in search of what Paul Erdős might have called *The Book*. He cries out,

> To me, it does not seem unlikely that on some shelf of the universe there lies a total book. I pray the unknown gods that some man—even if only one man, and though it may have been thousands of years ago!—may have examined and read it. If honor and wisdom and happiness are not for me, let them be for others. May heaven exist though my place be in hell. Let me be outraged and annihilated but may Thy enormous Library be justified, for one instant, in one being.

In the summer of 1985 I drove Paul to what many of us fondly remember as Yellow Pig Camp—a mathematics camp for talented high school students at Hampshire College. It was a beautiful day. The students loved Uncle Paul, and Paul enjoyed nothing more than the company of eager young minds. In my introduction to his lecture I discussed *The Book*, but I made the mistake of discribing it as being "held by God". Paul began his lecture with a gentle correction that I shall never forget. "You don't have to believe in God," he said, "but you should believe in *The Book*."

Paul Erdős Just Left Town

László Babai

"Ask Uncle Paul before you spend months on a problem." Thus János Komlós summed up his many years of experience with Paul Erdős, the easily accessible magic fount of knowledge.

Sadly, this recipe cannot be used anymore. Uncle Paul, who so cared for all of us, died of two successive heart attacks in Warsaw on September 20, 1996, while attending a graph theory workshop at the Banach Center.

The word spread like a brushfire; within a day most of the mathematical world knew. We gazed into our screens that brought the news, dumbfounded and struggling not to believe. Erdős had been a constant in our lives. He touched our minds as well as our hearts. His wry jokes about old age and stupidity had been around longer than most of us could remember; yet even at eighty he produced more papers per year than most of us do in a lifetime.

The mourners included Erdős's countless friends, coauthors, and all of us who had, time and again, observed his frail figure in conference lobbies across the globe, sunk in a chair, his mind open to link up with anyone interested in a mathematical problem.

Inimitable Style

A mathematician of unique style and vision, Erdős[1] will remain on the short list of those whose work defines the mathematics of the twentieth century. Erdős's interests covered a multitude of branches of mathematics. Foremost among them were number theory, combinatorics (including graph theory), set theory, classical analysis (especially the theory of interpolation), and discrete geometry, but his work extended to many other fields, including probability theory, topology, group theory, complex functions, and more, spreading over 40 percent of the two-digit classifications of the *Mathematical Reviews* [13].

With over 1,500 papers to his name, Erdős was the most prolific mathematician of our time. He considered mathematics to be a social activity: he wrote joint papers with more than 450 coauthors. A mathematical prophet of the jet age, Erdős maintained no permanent home base and was constantly on the move from coast to coast, continent to continent, to visit his ever-growing circle of disciples. He traveled with a small suitcase containing all his earthly belongings. "Property is nuisance," he used to say, paraphrasing the French socialists who thought property was sin. For most of his life Erdős had no regular income. When in 1984 he was awarded the $50,000 Wolf Prize (shared with differential geometer Shiing-Shen Chern), he promptly gave it away, keeping only $720 for himself.

Far from being a mathematical robot, Erdős was intensely interested in his human environment. He enjoyed classical music (which he called "noise"), he was well read in history, and he was informed about politics and society. Above all, he cared for the well-being of his friends and colleagues. "His money and his connections are available to promising students and mathematicians at all levels," wrote Ernst Straus in an eloquent tribute to Paul Erdős on the occasion of Erdős's seventieth birthday [17]. Erdős gave money to Ramanujan's widow (whom he never met), to relatives, colleagues, and students in need, including total strangers; he donated money to every worthy cause that came his way, which is amazing, given how little he had.

"How is the *epsilon*," Erdős would inquire about the baby of a friend. Another type of *epsilon,* ranking very high among Erdős's concerns, was fledgling mathematical talent. Erdős made connections with many a gifted high school student and maintained the contact through phone calls and correspondence. He suggested problems, solved and unsolved, for them to work on and was intensely interested in their progress. These *epsilons* were his children. His favorite *epsilon* was Lajos Pósa, whom Erdős helped to write an influential paper on Hamilton cycles when Pósa was thirteen!

Although Erdős spent less than 10 percent of his time in Hungary after 1934, many of his foremost protégés are from his native country[2].

Then again, Erdős's interest in mentoring and collaboration knew no boundaries. Here is a characteristic story: In Hawaii, a brilliant high school senior by the name of David Williamson proves that an odd perfect number (if there is one) must have exactly one prime factor that is congruent to 1 (mod 4). He asks his mentor, a university professor, whether the result is known. The professor does not know the answer but suggests to David that he write to Paul Erdős, who has just left town. The student does not need to wait long for the reply:

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[1]*Erdős's name is pronounced "air-dish"; the "i" in "dish" sounds like the one in "first".*

[2]*Cf. Erdős's 1971 article on "Child prodigies."*

Dear Mr. Williamson: (1985 IV 12)

Many thanks for your letter, which only reached me yesterday. The result you proved is in fact due to Euler. He also proved … Perhaps the following problem of mine will interest you. …

"This letter to a high school student won't rank very high on Erdős's list of accomplishments, but it did mean a lot to me," comments Williamson, now an IBM researcher and a rising star in combinatorial optimization.

H. Halberstam and K. F. Roth preface their monograph on *Sequences* (of integers) with this acknowledgment: "Anyone who turns the pages of this book will immediately recognize the predominance of results due to Paul Erdős. Insofar as the substance of this book may be said to define a distinct branch of mathematics—and its wide range of topics in classical number theory appears to justify this claim—Erdős is certainly its founder.…" The same can be said about an astonishing number of diverse areas of mathematics. Erdős's love of all these subjects resonates in the concluding words of the cited acknowledgment, shared by countless authors: "[Erdős's] unique insight and encyclopedic knowledge were, of course, invaluable, but the authors are no less indebted to him for his constant interest and encouragement."

A major way in which Erdős exerted his influence was in the open problems he posed. Straus writes: "In this century, in which mathematics is so strongly dominated by 'theory constructors,' he has remained the prince of problem solvers and the absolute monarch of problem posers."

With an incessant flow of elementary questions, Erdős breathed new life into a number of fields, including such seemingly dormant areas as Euclidean plane geometry, and helped create entire new disciplines, such as combinatorial number theory, Ramsey theory, transfinite combinatorics, extremal set theory, and the study of random structures.

Erdős's inimitable style of communicating directions of study in terms of interminable sequences of easily stated open problems helped focus attention on certain structures and phenomena more effectively than any philosophical pronouncements or project statements could have.

Erdős's childlike enthusiasm was contagious; it moved legions to attack his problems. Although Erdős never discussed the "big picture," it became evident to anyone who had some experience with problems Erdős disseminated (whether Erdős's own or someone else's) that they were pieces in a magnificent jigsaw puzzle; as a rule, both the results and the requisite techniques turned out to be profoundly relevant to large unexplored territories.

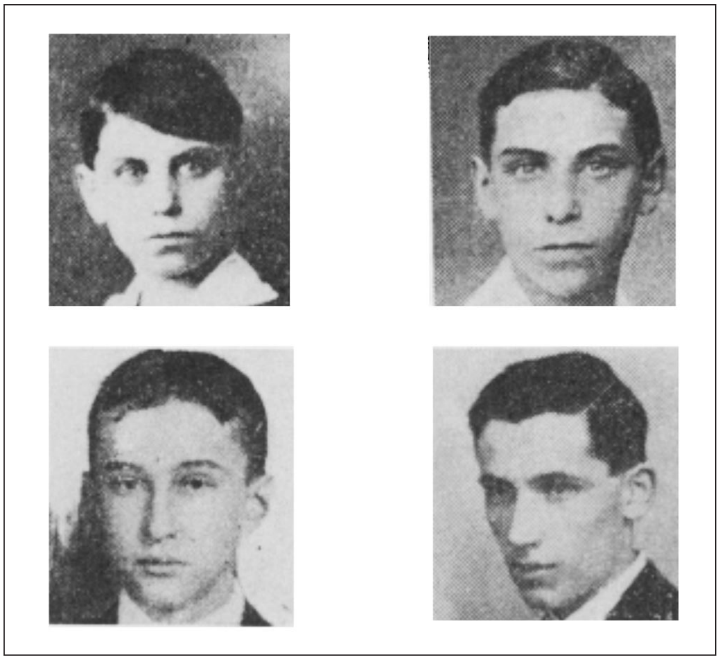
The *epsilon* is Ramsey theorist Jaroslav Nešetřil's son Jakub, near Prague, around 1980, with his father and with star Erdős-problem solver Vojtech Rödl (right).

The Wizard from Budapest

Paul Erdős was born on March 26, 1913, in Budapest to Hungarian-Jewish parents. Erdős's birth was marred by tragedy: his sisters (ages five and three) contracted septic scarlet fever and died within a day while Paul's mother was in the maternity ward. A year and a half later World War I broke out, and very soon Erdős's father was captured by the Russians and taken as a POW to Siberia for six years. By the age of four Paul was able to multiply 4-digit numbers in his head. He would ask visitors their date of birth and tell them how many seconds they had lived.

Both of Erdős's parents were high school mathematics teachers, and Erdős received much of his early education from them. Erdős always remembered his parents with great affection. At sixteen his father introduced him to some of his lifetime favorite subjects: infinite series and set theory.

During high school Erdős became an ardent solver of the problems proposed each month in *KöMaL*, the *Mathematical and Physical Journal for Secondary Schools*. Founded in 1893, this periodical is generally credited with a large share of Hungarian students' success in mathematics [15]. Erdős remained faithful to *KöMaL* and published several articles in it about problems in elementary plane geometry. At fourteen László Lovász came



Photograph courtesy of the J. Bolyai Mathematical Society.

In 1926 editor Andor Faragó started publishing the photographs of the best problem solvers of the *Mathematical and Physical Journal for Secondary Schools ("KöMaL",* Budapest), creating an invaluable archive. Top row: Erdős at 14 (1927) and at 17 (1930). Bottom row: Erdős' closest friends, Paul Turán at 17 (1927) and Tibor Gallai (aka Grünwald) at 18 (1930). *KöMaL* helped create the community of young math enthusiasts for over a century and is generally credited with a large share of Hungarian students' success in mathematics.

across one of these articles (1962) and was so enchanted that he read it "at least twenty times" and went on to become one of the most influential combinatorists of our time.

Back in the late 1920s it was on the pages of *KöMaL* that Erdős first encountered the names of his lifelong friends and collaborators, Paul Turán (1910–1976) and Tibor Gallai (then known as Grünwald) (1912–1992), among a large number of other young math enthusiasts.

As winners of national mathematics competitions, Erdős, Turán, and Gallai were admitted to Pázmány University, Budapest, in spite of the anti-Semitic "Numerus Clausus" law. The great analyst Leopold Fejér, professor at Pázmány University, Dénes König, professor at the Technical University of Budapest and author of the first monograph on graph theory (1936), and logician and number theorist László Kalmár from Szeged were major influences on young Erdős.

Under the shadow of the increasingly hostile social climate of the interwar period, Erdős and his friends escaped on weekly excursions into the hills of Budapest and discussed mathematics and politics. They forged friendships "that were the most lasting that I have ever known and which outlived…a vicious world war and our scattering to the four corners of the world," writes old pal George Szekeres in his moving foreword to The *Art of Counting* [4].

The political sympathies of this circle of friends lay with the left, including the underground Communists, the only organized force actively opposed to the extreme right menace. "L.A. is studying Jordan's theorem," Erdős would announce to his friends, having learned that their colleague László Alpár had been arrested (the prison walls represented the topological sphere in question).

Mathematics was the main subject in their conversations. With Erdős at the center these young people began to formulate exciting questions that would reverberate in a million forms throughout Erdős's work.

By the age of nineteen Erdős essentially completed his Ph.D. thesis, formally under Fejér. But the subject of Erdős's thesis was number theory; he proved the existence of prime numbers between n and $2n$ belonging to certain arithmetic progressions. Erdős's proofs were striking for their elegance. By the time Erdős graduated (1934) his name was known among the leading number theorists of the time. Issai Schur called him "der Zauberer von Budapest" [18]. Louis Joel Mordell arranged a four-year fellowship for him to Manchester.

Erdős spent the years from 1934 to 1938 in Britain on the Manchester fellowship. "His wanderlust was already in evidence," remarks Béla Bollobás [2]; "from 1934 he hardly ever slept in the same bed for seven consecutive nights, frequently leaving Manchester for Cambridge, London, Bristol, and other universities." During the Manchester years he was working mostly on number theory, but he also initiated work in combinatorics and Ramsey theory, most notably with Richard Rado (1906–1989), his lifelong friend and collaborator, a German-Jewish expatriate who had just escaped the Nazis. Notably the Erdős–Ko–Rado theorem, one of the key results in extremal set theory, was conceived during this period. The result became an instant classic upon publication twenty-three years later! At the same time Erdős maintained his mathematical contacts with his friends in Budapest, working especially with Turán and Géza Grünwald on analysis and with Gallai on graph theory.

Safe but Jobless in America

Erdős always listened to the news on the radio. On September 3, 1938, he sensed the imminence of the desperate times looming over Europe. He bade a hasty goodbye to his parents, left Hungary the same day, and soon left the Continent. He moved to the U.S. and did not return to the Old World for a decade.

In 1938–39 he held a fellowship at the Institute for Advanced Study with a stipend of $1,500 for the year. Even in 1995, more than 1,300 papers later, Erdős remembered 1938–39 as his best year.

The crop included two seminal papers, with M. Kac and A. Wintner, on the distribution of values of additive functions. The paper with Kac established a central limit theorem for a class of additive functions. “Thus with a little impudence we would say that *probabilistic number theory* was born,” Erdős wrote in 1995 [11].

In a mathematical adventure not uncommon for Erdős, he settled in short order the then outstanding unsolved problem of *dimension theory*: the (inductive) dimension of the set of rational points in Hilbert space. The experts expected the dimension to be zero or infinity, since this space is homeomorphic to its square. Erdős surprised the world by proving that the dimension is one.

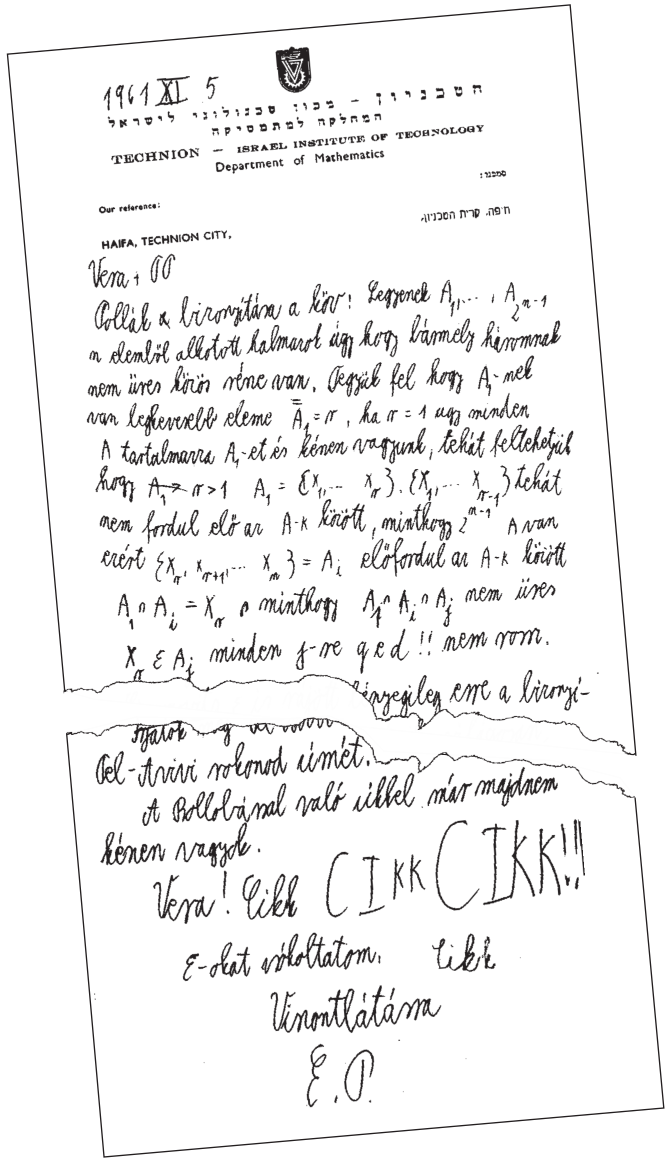
In spite of these and other major results, Erdős’s fellowship at the Institute was not renewed, and subsequently he was without a job for considerable periods of time and survived on small loans from colleagues. His financial situation improved temporarily with a “research instructorship” at Purdue in 1943, but after 1945 he was without a job again. Meanwhile, the flow of groundbreaking results continued unabated. Erdős’s paper “On the law of the iterated logarithm” appeared in 1942; the first study of “inaccessible cardinals,” fundamental to modern *set theory*, saw light in a paper by Erdős and Tarski in 1943, and the Erdős–Stone theorem, which opened up the field of *extremal graph theory*, appeared in 1946.

Throughout the war Erdős was in continual anguish over the fates of his parents and friends. Erdős’s father died of a heart attack in Budapest in 1942. His mother fell into depression. There was no postal service between the U.S. and Hungary; Erdős tried in vain to contact his mother through intermediaries.

Hitler’s forces descended upon Hungary on March 19, 1944. Within weeks the clockwork of the “Final Solution” engulfed Hungarian Jewry. Among those who perished were most of Erdős’s relatives, including four of his mother’s five siblings; many of Erdős’s young friends, including Ervin Feldheim and Géza Grünwald, Erdős’s early coauthors on the theory of interpolation; and also Dániel Arany, the founder of *KöMaL*, and Andor Faragó, the legendary editor and publisher of *KöMaL* through the interwar period. Dénes König, who had introduced Erdős and the world to graph theory, committed suicide when ordered by the janitor to move to the ghetto.

As if by a miracle, Erdős’s beloved mother survived, as did Erdős’s intimate friends Turán and Gallai. Another survivor was Vera Sós, Gallai’s brilliant high school student. Sós later married Turán, and two decades later she became one of Erdős’s foremost collaborators in combinatorics.

In July 1948 Erdős met Atle Selberg at the Institute for Advanced Study, and from their brief encounter an elementary proof of the Prime Number Theorem emerged. This result was prominently mentioned in Selberg’s Fields Medal citation in 1950 and in Erdős’s Cole Prize citation (AMS) in 1951. It is a sad note on the history of number theory that a controversy over the genesis of this seminal work prevented these two great mathematicians from further collaboration.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Department of Mathematics
Our reference:
HAIFA, TECHNION CITY,
חיפה, קרית הטכניון

Vera, TP
Pollák a bizonyítása a köv: Legyenek $A_1, \ldots, A_{2^{n-1}}$
n elemből álló halmazok úgy hogy bármely háromnak
nem üres közös része van. Tegyük fel hogy A_1-nek
van legkevesebb eleme $\bar{A}_1 = r$, ha $r = 1$ úgy minden
A tartalmazza A_1-et és készen vagyunk, tehát feltehetjük
hogy $A_1 \ldots r > 1$ $A_1 = \{x_1, \ldots x_r\}$. $\{x_1, \ldots x_{r-1}\}$ tehát
nem fordul elő az A-k között, minthogy 2^{n-1} A van
ezért $\{x_r, x_{r+1}, \ldots x_n\} = A_i$ előfordul az A-k között
$A_1 \cap A_i = x_r$ minthogy $A_1 \cap A_i \cap A_j$ nem üres
$x_r \in A_j$ minden j-re qed !! nem rossz.
... lényegileg erre a bizonyí-
...
Tel-Avivi rokonod címét.
A Bollobással való cikkel már majdnem
készen vagyok.
Vera! Cikk CIKK CIKK!!
E-okat üdvözlöm. Cikk
Viszontlátásra
E.P.

Erdős’s correspondence was phenomenal. This sample letter was written in 1961 at the Technion (Israel), where Erdős held the title of a “permanent visiting professor.” The letter, written in Hungarian, is addressed to Vera Sós and her husband, Paul Turán. *Vera and TP, Pollák’s proof runs as follows: Let* $A_1, \ldots$ The second page describes a proof by Rado. In the concluding paragraphs Erdős asks Turán to provide the address of a relative, reports that an article with Bollobás is nearly done, and emphatically urges Vera to finish an article (CIKK = article). Closing: *Give my love to the* ϵ*-s* [a reference to the two children of Sós and Turán], *Good-bye, E.P.*

Erdős Maxims

Another roof, another proof.

Property is nuisance.

L. A. is studying Jordan's theorem[1].

Neither Sam nor Joe can determine when and where I should travel.

Sam and Joe went up the hill[2]...

The S. F. stole my passport.

Is this lovely *epsilon* a boss[3] or a slave?

My first two and a half billion years in mathematics[4].

Do not count articles, weigh them.

The S. F. has a transfinite book of theorems in which the best proofs are written. You don't have to believe in God, but you should believe in *The Book*.

[1] This one *really* does not mean what it seems. See the article.

[2] Erdős points out that Jack and Jill were politicians in the Elizabethan time.

[3] Boss = female, slave = male. In spite of the superficial appearance of gender bias in his language, Erdős was a genuine "equal opportunity" friend and collaborator.

[4] The earth is believed to be 4.5 billion years old. At school Erdős had been taught that it was 2 billion.

Freedom over Convenience

Erdős left the U.S. in 1948 for the first time in a decade and began what would become an unending journey around the globe. In Amsterdam he met a childhood acquaintance, Alfréd Rényi (1921–70), eight years Erdős's junior, who had meanwhile emerged as a mathematical genius working in a great variety of fields, including number theory, probability theory, orthogonal series, information theory, combinatorics, and applied mathematics. He was the exact right match for Erdős. The two began their joint explorations with a paper "On consecutive primes," one of Erdős's lifetime favorite subjects.

In autumn 1948 Erdős briefly visited Hungary. He rejoiced with his mother and his old friends who had survived.

By 1949 the Soviet-orchestrated conversion of Hungary into a communist state was in full swing, the borders hermetically closed, and Erdős was again unable to visit home for several years.

In 1952 Erdős finally landed a secure job at Notre Dame University. He lost that job two years later to McCarthyist paranoia: because of his connections to communist countries (number theorist Hua in China, Erdős's mother in Hungary) and his refusal to condemn Marx, the Immigration Service denied his reentry permit. Erdős could have stayed in the States, but he chose freedom over convenience, attended the International Congress of Mathematicians in Amsterdam without the reentry permit, and was unable to return to the U.S. for nine years (except for a brief visit on a "special visa"). In 1963, when Erdős's name was finally removed from the list of "undesirable aliens," he informed his audience at an AMS meeting with his characteristic humor that "Sam[3] finally admitted me because he thinks I am too old and decrepit to overthrow him."

"It is a sad commentary on our time and country that this man—so totally immersed in scholarly work, so remote from the political arena, a free spirit who lives by the highest moral standards—could be harassed by bureaucrats in high position whose duty is to protect our freedoms," wrote Michael Golomb in the May 1977 issue of *Science*, commenting on the incident that deprived the U.S. mathematical community of the presence of Erdős for nine years and Erdős of his last regular job and his permanent-resident status in the U.S.

Once Erdős set foot on American soil, he took the country by storm. He acquired new friends at a phenomenal rate. At the 1963 meeting he met Ron Graham, who soon became one of his closest collaborators in number theory and combinatorics and for decades provided Erdős with a home base in New Jersey. A decade later Erdős was adopted into the family of graph theorist Ralph Faudree in Memphis; Faudree's home became a fixed point on Erdős's meandering trail, and Faudree became Erdős's prolific coauthor on Ramsey-type problems in graph theory.

Back in 1954 a job in Israel "saved [Erdős] from starvation," as he used to recall with gratitude. In 1955 the Technion appointed him a "permanent visiting professor," a title he held until his death. After this appointment, he was listed in his passport as a resident of Israel, while maintaining his Hungarian citizenship. He forged close ties with the mathematical community of Israel. In 1976 he founded an award in Israel in memory of his parents to honor the accomplishment of young mathematicians. (He founded a similar award in Hungary in 1972.) The award carries great prestige. From his Wolf Prize (1984) Erdős endowed a postdoctoral fellowship at the Technion to commemorate his mother. Leading combinatorist Noga Alon first met Erdős at the Technion as a college freshman in 1975. Three years later Erdős told him a problem which became the subject of his M.Sc. thesis. "My very first paper was on an extremal problem suggested by Erdős; the first serious book in combinatorics I read was *The Art of Counting* [4]. … I suppose I would have been a mathematician even if I had never met Erdős, but my work would have surely been totally different," says Alon.

In 1957 Erdős became affiliated with the Mathematical Institute of the Hungarian Academy of Science, another permanent affiliation he would maintain for the rest of his life. Although Hungary remained under communist rule for three more

[3]"Sam": the U.S. in Erdős-ese. "Joe" was Erdős's nickname for the Soviet Union and for communist countries, referring to Joseph Stalin.

decades, after 1956 Erdős gained the singular privilege of being able to visit Hungary *and leave*. This allowed him to initiate and maintain several of his most prolific collaborations. With Rényi he created a fascinating new synthesis of combinatorics and probability in their landmark study "The evolution of random graphs" (1960, 1961). With Turán he studied *interpolation* and invented *statistical group theory*. With Hajnal he expounded *combinatorial set theory*. At a hillside resort of the Hungarian Academy of Science he enjoyed his mother's company and turned the place into a mathematician's Mecca; he worked simultaneously on diverse subjects with all his visitors.

Erdős's uncompromising view on freedom and dignity compelled him to take a voluntary exile from Hungary in 1973 after the Hungarian government denied visas to Israeli mathematicians, including old friends of Erdős, who wished to attend a conference held in Hungary to celebrate Erdős's sixtieth birthday. "Joe is being trivi[4] again," Erdős would declare, expressing his contempt. Erdős lifted his self-exile three years later to attend the deathbed of his dear friend, Paul Turán.

With his legendary suitcase Erdős circled the globe several times a year. He spent most of his time in the U.S., Canada, Hungary, Israel, the U.K., France, and The Netherlands, but visited many other countries around the globe with fair frequency, including a number of visits to Australia. His mother accompanied him on his journeys from 1964 until her death in Calgary in 1971 at the age of ninety-one.

After his mother's death Erdős became depressed, and he fought this by putting in 19-hour workdays. He continued to produce important results and conjectures at ever-increasing rates and initiated joint work with ever-new generations of mathematicians. Among the many to whom Erdős provided a thesis topic was Joel Spencer; Joel reciprocated by editing the *The Art of Counting* (1973) [4] and writing a book with Erdős entitled *Probabilistic Methods in Combinatorics* (1974) [9]. The well-chosen topics of these volumes and the artful exposition of [9] were instrumental in bringing Erdős's world of finite combinatorics to a wider audience. Noga Alon says, "I took notes of most of the papers in [4], and I often find myself using results from there even in these days."

Although most of Erdős's work was in combinatorics and number theory, he never abandoned other old favorites of his. In 1980 he published a spectacular theorem with Vértesi on interpolation: Given *any* system of nodes, there exists a continuous function f such that the sequence of Lagrange interpolation polynomials of f on the given node sets diverges almost everywhere.

During the last decades of his life Erdős was showered with honors. He received at least fifteen honorary doctorates. He became a member of the national scientific academies of eight countries, including the U.S. National Academy of Sciences (1979) and the Royal Society (1989). Shortly before his death Erdős renounced his honorary degree from the University of Waterloo over what he saw as unfair treatment of combinatorist J. A. Bondy.

"He died with his boots on," remarked Ron Graham. Two days before his death he gave a splendid talk at the Banach Center. A day later he finished his last single-authored paper. With a ticket in his wallet he was ready to move on to the next conference: number theory in Lithuania.

By tragic coincidence, none of his close friends from the Banach Semester Erdős attended was in town at the time of his last struggle with the S.F.[5]

We do not know his last words. But his legacy is enormous. It lives on in the minds of his disciples, in his innumerable proofs and conjectures, in exciting gaps between upper and lower bounds, in tantalizing derandomization challenges presented by probabilistic proofs of existence, in our

Reflections on Erdős

I am the *epsilon* from Szeged.

—14-year old math whiz Attila Máté introducing himself to Erdős's mother

Children become letters.

—Erdős's mother

Mathematicians become *Collected Papers*.

—Erdős, paraphrasing his mother

A surprising application of number theory.

—Paul Turán about his 1935 joint paper with Erdős published in Tomsk (Soviet Union). Turán used a reprint as his sole ID when a Soviet military patrol stopped him in the streets of liberated Budapest in 1945. The patrol was impressed, and Turán was saved from a trip to the GULAG.

I read the article at least twenty times.

—László Lovász about an article by Erdős on plane geometry in the Mathematical Journal for Secondary Schools. *Lovász was 13 at the time.*

I learned enormous quantities by collaborating with many members of the Erdős School.

—Avi Widgerson, Nevanlinna Prize laureate

In the search for truth there is room for Don Juan's like Erdős and Sir Galahad's like Einstein.

—E. G. Straus, who had worked with both Einstein and Erdős

Regardless of the sediments washed up around his figure by the ever-growing flow of legends, his brilliance in mathematics, the purity of his character, and his compassionate, hands-on humanism will clearly and irrevocably stand out.

—Vera Sós

[4]*Trivi: truncation of "trivial." This Erdős-ese term translates roughly to "small-minded," "mean-spirited" (person, government, etc.).*

[5]*S.F.: Supreme Fascist, Erdős-ese for the God who sends us flu, misplaces our passports, and hides the pages of* The Book *from us.*

Erdős's mother, Anna (1880–1971), joined her son on his journeys around the globe at age 84 and travelled with him everywhere until her death at 91. This photo shows mother and son in Melbourne, 1969.

own asymptotic questions, and in memories of a kind and generous human being who lived on a web of trust[6], who, by his example, taught us trust and compassion.

Recommended Reading

[4] is a selection of Erdős's work in finite combinatorics. [18] is a profound yet enjoyable survey of all areas of Erdős's early work except set theory. It was written in 1963 by Paul Turán, Erdős's soul mate in several areas. [16] and [12] are two pairs of recent volumes containing a wealth of material on Paul Erdős and his mathematics. Both [16], vols. 1–2, and [12], vol. 2, contain up-to-date (at the time of publication) bibliographies of Erdős. A very special paper by Erdős appears in [16], vol. 2, on his favorite theorems [11]. For a biographic study of Erdős we refer to [1]. [12] is a thoughtfully edited pair of volumes containing a number of superb survey articles on Erdős's work. The highlight of the volume is an article by Béla Bollobás, who summarizes Erdős's mathematical spectrum in thirty thoroughly enjoyable pages, in addition to providing eight pages of biography [3]. András Hajnal, Erdős's No. 1 collaborator, contributed an informative article to [12] on Erdős's set theory, peppered with revealing stories on four decades of prolific collaboration [14]. [12] also includes an entertaining article with a variety of statistics on Erdős's work and collaborations [13].

The bibliography below includes the books and book chapters written by Paul Erdős. [10] is a wonderful volume which captivated this writer at 16.

A video documentary on Erdős entitled *"N Is a Number—A Portrait of Paul Erdős,"* by George Paul Csicsery, is available from the MAA.

For more references we recommend the Web page `http://www.cs.uchicago.edu/groups/theory/erdos.html`.

[6] *Phrase coined by Joel Spencer.*

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