CHESS AND MATHEMATICS

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Is chess a junior brother of mathematics? Are there similarities between chess and scientific life in Soviet Russia? Chess is played by two partners on a board of 8 x 8 squares. They marshal the white and the black armies of 16 pieces each: pawns, knights, bishops, rooks, a queen and a king. First, White makes a move, then Black and so on. Different pieces move in different ways; a move may include the capture of an enemy piece. The purpose of the game is to capture (kill) the enemy king. If this is impossible, or if partners consent, the game is a draw.

Chess appeared in the Islamic world at the time of Islam's flowering in the 8-10 centuries A.D. as shataranj—"the king of games and the game of kings." The pieces were as we know them today but their rules were very different. At the end of the 15th century in the Western world, queens, bishops and pawns became more mobile, making them more powerful. Western chess is much more dynamic and faster than its Eastern predecessor. Chess is an intellectual game, excluding a component of chance. A player is required to evaluate each of his positions and to compute a series of moves to improve them. A position's value depends on the number and quality of the remaining pieces on the board and on its dynamics, its possibilities of future development. Chess has become international: most countries have national championships; international tournaments are popular. It is estimated that the volume of chess literature exceeds that of all other games combined.

Tsarist Russia, strong in chess before the Revolution, lost several of the world's leading players when its borders changed in the Civil War. Some, like Alekhin and Bogolyubov left the Soviet Union soon afterwards for the West.

It was the chess enthusiast and Communist Party member A. F. Ilyin- Zhenevskii who started to organize Soviet chess with the intention to provide factory workers with a meaningful intellectual pastime and to lure them away from legally forbidden games of hazard. Even for bridge, public competitions were not tolerated in the Soviet Union. Around 1927–28, Soviet players were middle-aged amateurs, inferior in strength and experience to foreign grandmasters. The best, G. Ya. Levenfish (1889–1961), P. A. Romanovskii (1892–1964) and F. P. Bohatirchuk (1892–1984) were employed, respectively, as an engineer, a financial specialist, and a doctor of medicine and director of a radiology institute in Kiev.

The state of affairs was not satisfactory for the ambitious N. V. Krylenko (1885–1938), who led Soviet chess since 1924. Commissar of War in the first Bolshevik government and a public prosecutor at several show trials, his importance in the Party steadily declined until he perished in a purge. But during his life, as president of the chess section of the All Union Council of Physical Culture, he had done much for chess. He had organized three international chess tournaments in Moscow for the world's best players. The popular Soviet film, "Chess Fever," with the charismatic Cuban world champion Capablanca as its main hero, featured the first of the tournaments.

Supporting these developments, many excellent books, journals and newspaper columns invited the study of the game in Soviet Russia. Chess schools encouraged youngsters. Chess clubs appeared everywhere. Chess became the most popular sport, leaving behind soccer. Players of a new generation began to mature around 1931; some of the best of them were raised by Romanovskii in Leningrad. Neither they nor the members of the old guard were good enough for the Party and Krylenko, whose ambitions were set very high. They reached for nothing less than Soviet players becoming the world's best.

Only the formidable M. M. Botwinnik (1911–95) was an exception. Undisputed first in the 1931 and 1933 Soviet championships, he had established his world class in competition with world champions Alekhin, Capablanca, Euwe and Lasker. In Leningrad in 1934 he was first ahead of Euwe; in Moscow in 1935, first with Flohr, ahead of Lasker and Capablanca; in Moscow in 1936 second after Capablanca, ahead of Lasker; and most importantly, at Nottingham in 1936, first with Capablanca, ahead of almost all of the chess elite, including Alkhin, Euwe, and Lasker.

Never were Soviet chess players persecuted for their way of playing, sharing this advantage with other sportsmen and actors of theater. In contrast, a scientist or educator could endanger himself by his "ideologically incorrect" science or technology, or by giving a "bad example" for his students. The chess bureaucracy often showed preference to players who were considered potentially more promising, for instance being younger. The 1931 championship was the best example. It was played in eight preliminaries, with two win-

ners from each qualifying for the final. Romanovskii advanced, but the very gifted Moskovite N. N. Ryumin (1908–1942) did not. Dissatisfied, Krylenko arbitrarily extended the final, including Ryumin and two former champions who did not play in the preliminaries. Romanovskii resigned in protest. Although Ryumin placed second in this championship, the fame of both stars faded afterwards. Levenfish tried to improve his game hoping to become a chess professional. He won the next two Soviet championships in 1934/35 and 1937, where Botwinnik did not participate. Defending his title successfully in a drawn match against the latter, he became the second Soviet grandmaster after him. Temporarily (on paper) he was ahead of Botwinnik. Nevertheless, he was not granted his ardent wish to compete abroad. He never received a grandmaster's stipend and died in abject poverty [5, p. 201].

A second wave of youthful Soviet talent started to produce new grandmasters around 1939. In addition, excellent players were acquired through the extension of Soviet borders: Keres and Tal from the Baltic republics; others, like Flohr, from Central Europe where National Socialism was on the rise. Moving in the opposite direction was rare because of the powerful governmental support of the best Soviet grandmasters. The most spectacular case was the emigration in 1976 of V. L. Spassky (1937 –) and B. V. Korchnoi (1931 –). The German occupation of Kiev in 1941 was used by Bohatirchuk to become a fugitive from Stalin. Trying to move as far west as possible, he joined the medical research institutes in Krakow and in Berlin. In May 1945 he reached the American Zone of occupied Bavaria. For a time he played in German chess events under the pseudonym Bozhenko, "to throw Soviet investigators off the scent" [2, p. 42]. He immigrated to Canada and found occupation in his specialty in Ottawa. As a curiosity, he had a score of 3: against Botwinnik in their personal encounters.

The most important international tournament of the 1930s, organized in 1938 by the broadcasting company AVRO of Netherlands, made clear who were the most likely candidates for the world championship crown of the aging Alekhin. P. Keres (1916–1975) from Estonia and the American R. Fine (1941 –) were the joint winners; Botwinnik was third. Alekhin, the American S. Reshevsky (1911 –) and Euwe shared the fourth place achieving 50Capablanca and Flohr lingered behind with negative scores.

During WWII chess life in the West was dormant. Keres and Alekhin, the latter regularly the better of the two, played in several tournaments in Germany and Czechoslovakia. They were joined by the brilliant young K. Junge (1924–1945), an officer of the Wehrmacht, killed in action in April 1945. Alekhin, who lived in Paris, did not behave ethically during the war, publishing in the Nazi press three slanderous anti–Semitic articles about his fellow grandmasters of Jewish origin. From 1943 to 1946 he lived in Spain and Portugal in poverty and disgrace. His overtures for a match with Capablanca were disregarded. After the war the Soviet government arranged a match for him and Botwinnik to be played in London. He died in March 1946, a day after receiving a final invitation. It is essential for our story to note that chess prospered in the USSR during WWII, except for the war's darkest years.

For Western Europe WWII started in 1939, for the Soviet Union only in 1941. Meanwhile Soviet chess continued to develop, with many new Soviet grandmasters. The "absolute Soviet championship" of 1941 showed Botwinnik's mastery over them. In 1939 and 1946 two efforts were made to arrange a match for Botwinnik with the world champion Alekhin. They failed; the first at the war's beginning, the second because of the champion's sudden death.

Botwinnik possessed determination, fearlessness, excellent sportive discipline and deep strategic insight. He was the first to put his preparations for chess contests on a scientific basis, practicing with his sparring partners in rooms full of tobacco smoke and disturbing noise. He prepared chess openings for one-time spectacular wins but most importantly developed rich opening systems useful for him for years. World War II ended not only with Soviet victory over Germany and with Soviet chess victory over the whole world. Its chess superiority was established in the 1945–46 radio matches against the best players of Great Britain, won with the score 18:6, and of the USA, with 15:4. Life conditions for Soviet grandmasters became very good: they were awarded stipends, granted individual trainers during a contest, had seconds helping them with analysis and preparation for their approaching parties. At tournaments adjourned positions of a day would often be analyzed by a team of the friends of a player—a practice not forbidden, but considered unethical by older champions. The remuneration for success in contests remained modest. It was assumed that winners should be proud and eager to serve the state and nation in this way, regardless of material considerations.

After Alekhin's death in 1946, the International Chess Federation (FIDE) took the determination of a new world champion in its hands. A match-tournament of the five best of the 1938 AVRO tournament should

solve this. But a problem arose. Fine had a phenomenal trip to Europe in 1936–38, winning everything on his way. During the war, however, Fine developed other interests. He successfully studied psychology—a profession that promised a much better material life than the meager income of a world chess champion. The young Russian Smyslov replaced the resigning Fine and the Soviet government "pardoned" Keres for playing in German tournaments during the war.

Botwinnik won the match-tournament handsomely. The period 1946-63 can be called Botwinnik's era of world chess, when he withstood numerous attempts to dethrone him. The dominance of Soviet grandmasters became so prominent that Botwinnik never had to face a foreigner in his world championship matches. His success was astonishingly mediocre. He drew with difficulty his first match with Bronstein, then drew, lost and won three matches against Smyslov, lost and won against Tal, finally lost in 1963 against Petrosyan. He did not continue his efforts since FIDE had cancelled the automatic right of a champion to a return match after a loss. This produced two Soviet ex-world champions, Smyslov and Tal, soon to be joined by Petyrosyan and Spassky. Together with matches against Flohr in 1933 and with Levenfish, of the nine highest level matches Botwinnik drew four, lost three, and won only two for a total score of +43-46=92.

Botwinnik's misfortune may have been his wish to be a model Soviet sportsman, in particular to be an amateur, have a viable profession outside of chess. His studies of electrical engineering at the University of Leningrad earned him shortly before the beginning of WWII a postgraduate degree of doctor of technical sciences. This degree, much higher than a Ph.D. degree in the USA, was normally both required and sufficient for a full professorship in the Soviet Union. Botwinnik did not engage in teaching or research typical of a professor's activity. His interest remained chess. Only after his retirement did he engage in related activity trying to work out a program for a chess playing computer.

Botwinnik, who won seven Soviet championships, ranked first in the world in the 1940s; later he proved to possess a strong survival power that he could apply when necessary. No wonder that he was called the "Patriarch of Soviet Chess." He himself thought to be only "first among equals." Successful technologists who were of grandmaster strength included M. Euwe, M. Vidmar and P. F. Bohatrchuk. In general, being a professional in chess does not allow success in other professions. Several Soviet grandmasters had a complete university education, but usually in easier subjects: Tal in journalism, Korchnoi in history.

There are many styles to play chess. Positional play consists of maneuvers, based on general considerations, made with the aim to improve a player's position. It requires a fine feeling for positions that arise, for their permanent properties, for their dynamic possibilities. In positional play one tries to win by accumulating small advantages. In contrast, combinatorial play is a series of violent, often paradoxical moves with the purpose to mate, to gain material, or simply to improve one's position. Many combinations contain a sacrifice of material. Botwinnik requires this even in his definition of a combination. However, Fischer often would do the opposite, sacrificing his opponent's pieces rather than his own. Snatching his partner's pawn, he would put himself in a very dangerous position and only his following combinatorial play would establish his advantage. One can say that positional play is akin to strategy in war, while combinations correspond to tactics. The first world champion Steinitz and his pupil Tarrasch taught that in a better position one often has to attack, use combinatorial means, otherwise the advantage may disappear

Best players are universal, they know how to combine and use both modes of play. Examples are the great champions Fischer and Kasparov. With others, one side predominates: for Alekhin combinations, for Karpov positional play. Grandmaster, R. Spielmann said that in the positions from Alekhin's games, he could find the combinations, but didn't know how to reach his positions. The English mathematician G. F. Hardy said once that mathematics is greater than chess where one can sacrifice only a queen, while in mathematics, in proofs by contradiction, one sacrifices the whole game and still wins by proving that the opposite of the theorem to be proven leads to a logical absurdity. Beauty in chess manifests itself in an unusual combination or in the logical exploitation of positional advantages. Often beauty is hidden. A game follows a simple positional line except that the player's opponent seems to overlook a promising, superior possibility. In reality, this line would be refuted by a hidden combination. Here, beauty lies in what does not appear on the board.

The Soviet press used to proclaim the existence of a special Soviet school of chess. Combinations were praised as its preferred features. Actually everybody could play as he deemed best. There was no pressure on Soviet players in this respect. Freedom is an essential component of the success of an intellectual enterprise.

Fischer or "One against the Soviet Union"

Can a single individual prevail against a determined sport organization of a powerful country and emerge an overwhelming victor? The American Robert James "Bobby" Fischer (1943 –) achieved exactly this. He was the son of immigrants—a Swiss–Jewish mother and a Hungarian–Jewish father. Bobby learned to play chess when he was six; in the fertile chess environment of New York his mother found for him chess mentors and friends he needed for his development.

When he was 13, the tall boy in simple clothes attracted popular attention by his spectacular play in chess events of the country. Soon he could beat chess masters. The game against one of them, D. Byrne, earned the prophetic and extravagant name THE GAME OF THE 20TH CENTURY. In a seemingly unsatisfactory position Fischer launched an attack, sacrificed his queen. In a 13 moves long combination (a move in chess means a double move, one move by each, White and Black) he regained more than he had sacrificed, enabling him to mate the opponent's king. There are many chess combinations deeper, more complex than this one; Fischer did not include it in his classic book [2] but his was a worthy introduction into the important, spectacular, unbelievable chess life of the 20th century.

Chess for Fischer was always identical with his life. Very soon he left school saying that his instructors could not teach him anything useful. In the service of pure chess, he grew up an introvert and self-centered. He became asocial, more often than not misinformed about matters other than chess. He turned into a self-centered man who was also a child. His great rival Spassky whom he vanquished in 1971 called Fischer's play childish. And this was true. Fischer's play was based on sound and transparent positional considerations that appeared simple when shown. Astonishing combinatorial turns were their organic components—another side of Fischer's genius. Always trying to win, avoiding draws, he would often hazard into dangerous positions, but made fewer errors than his adversaries. Here is what the contemporaries of the mature Fischer said about him: "He has deep strategic mastery and filigrane technique" (Tal); "he has unusual combinatorial ability" (Botwinnik); "the fantasy and logic of his play produce its artistry and its sporting aspect" (Smyslov) [7, p. 430].

In December 1957, Fischer, at age 14, was invited to his first serious tournament, the United States championship. He won it without a loss, ahead of the world-class player Reshevsky. This was the first of several forward jumps for Fischer. Even at this time, he thought of nothing less than world championship and told so to anybody who would ask him. This win offered him direct access to it. New FIDE's world championship cycle of that time, repeated every three years, consisted of the large Interzonal tournament of national champions selected world-wide. American championship provided one of them. Next, the first six winners of the Interzonal together with two older champions played a match-tournament of candidates, each contesting four games against each other. Finally, the winner of this incredibly strong event could confront the reigning word champion in a match.

Bobby had the right to play in the 1958 Interzonal in Potoroz, Yugoslavia. Asked why he hoped to qualify there to the next stage, without international experience and against the world's best players, Fischer replied: "I can draw with grandmasters and there are half-a-dozen patzers I reckon to beat." Both problems proved to be difficult for Fischer. Initially, he succeeded to draw with Tal and Petrosyan, and defeated Larsen brilliantly. After this, other grandmasters, perceiving him dangerous and unpredictable, left him alone. And still Fischer needed luck. He squeezed among the six best by the skin of his teeth when grandmaster Bronstein, ahead of him, lost to an outsider. Barely 15, Fischer was a grandmaster.

A year later, in the match-tournament of the candidates, Fischer could not withstand the top Russians. He lost to Tal, 0:4, to Petrosyan, 1:3 and was level 2:2 with Keres and Smyslov. In the last game with Tal Fischer began strongly, having for many moves a won position until Tal recovered to win. "He will not escape me the next time," promised Fischer, and he was right. In 1959-60, Fischer played in many international tournaments with success and sometimes less well. In the strong Buenos Aires 1960 he finished with an unbelievably low score for him, the worst in Fischer's life, +2=11-5.

Fischer's next jump was from grandmaster to one of the best in the world. In the 1961 Bled tournament he was second after Tal without loss, +8=11; at the Interzonal 1962 in Stockholm he placed first with +13=9, each time with four Soviet giants participating. In Bled, Fischer's goal was to defeat all the Russians, and he almost achieved it. He beat Tal (his promised victory), Geller, and Petrosyan and drew against Keres. In Stockholm, he played with elegance and ease for a high score, 2 points above the rest. The Soviets were not yet ready to confront him. They were preparing for the next elimination step in 1962 in Curaço in the

Netherlands Indies. Fischer firmly expected to gain the right to play Botwinnik, but he suffered a bitter disappointment. He had a bad beginning and his wonderful recovery ability did not work this time. He lost two parties each to his bê te noire Geller and to his admirer Korchnoi. Against Petrosyan he had 1:2 which could not prove that the latter's loss to him in Bled was anything more than an accident. Behind the winner Petrosyan, Keres, and Geller, Fischer ended fourth, scoring +8=12-7. Next year Petrosyan beat Botwinnik. He and not Fischer became the new world champion.

What was the explanation of this fiasco? After the tournament, in an article in Sports Illustrated entitled "The Russians had fixed world chess," Fischer claimed that at Curago the Russians played as a team, at full strength against him and with easy draws among themselves. He claimed that they were analyzing his positions together, even commenting on them in Russian while he was playing. After defecting to the West fourteen years later, Korchnoi revealed that a conspiracy against him and Fischer did exist between the first three prizewinners. The Yugoslav grandmaster Gligoric commented, "Five Russians cannot be weaker than one Fischer." According to general opinion Fischer at 19 was still too young to win the grueling contest. In any case, FIDE indirectly agreed with Fischer's accusations. Henceforth match—tournaments of the candidates were replaced by eliminating matches between them.

Another disappointment for Fischer was his drawn game with Botwinnik at the Varna Chess Olympiad of 1961, where both met on the first boards of their countries. Botwinnik described it very frankly: "To my great shock, Fischer refuted overboard a variation which I have analyzed at length. He won a pawn with a better position. The game was adjourned in a rook endgame. With my advisers and friends, we analyzed the position all night, hoping to find a draw. Finally, Geller found a very fine idea. Unexpected by Fischer, next morning it yielded the desired draw." From these incidents Fischer derived a conviction that the Russians are his sworn enemies and that only his own superior force can overcome them [4]. Times have changed since the 1920s and 1930s when Capablanca, Lasker and others were considered superior friendly talents who could help the Soviets to improve their chess abilities. Now Russian players themselves had to be the best. From October 1962 to August 1965 Fischer boycotted chess, in particular tournaments organized by FIDE. It was even rumored that Bobby abandoned chess for good. However, he considered it his duty to participate in the 1962-63 and 1963-64 United States championships. The results showed that his strength continued to increase. In the second event he achieved a "clean win" without draws and losses. Clean wins are extremely rare and we know now that his score of +11=0-0 was achieved at this level only once in the 20th century. In Greek mythology the giant Atlant, son of Earth, gained strength each time he was thrown to the ground. Like him, Fischer gained strength after recuperating from a mishap.

In preparation for the 1967 Interzonal in Sousse, Tunis, Fischer played in two European tournaments with good Russian participation. In the Interzonal, a monstrous all-play-all tournament with 22 participants, Fischer played brilliantly, fearlessly and with excellent preparation. He reached a lead with +7=3-0 or 8:1. Particularly important for Bobby was his attacking game against Shtein, the 1967 Soviet champion. At the summit of his short career, Shtein (1934-1973) had just won Moscow's tournament to honor the 50th jubilee of the "Great October Social Revolution." All leading Soviet grandmasters, except Botwinnik and Korchnoi, had played in that tournament.

In Sousse, Fischer refused to play on Sabbaths—he had joined a small Christian sect—and was falling behind the tournament's schedule. Three of his games were declared forfeit and Fischer simply resigned the tournament. By this time he had played 10 games; 11 were still outstanding. An examination of the tournament table reveals that in order to end as one of the six winners, it would have been sufficient for Fischer to make five points in these 11 games. He could also have played some of the games with reduced attention. But this was against his principles. Fischer's quest for world championship was delayed by another three years. Once again, he was his own worst enemy.

Fischer again dropped out of chess. In the 2 years, from October 1967 to March 1970, he played with great success in two relatively weak tournaments in Israel and Yugoslavia—two countries where he was very popular. Later Fischer explained that he had been hiding from publicity to better prepare his revenge. Meanwhile, he created more obstacles for himself. Bobby's seventh in row win of the USA championship was narrow this time; he lost games to Reshevsky and R. Byrne. In no tournament was his victory absolutely sure! Fischer threatened to abstain from the championships unless they were made "more serious," longer than 11 rounds. This wish the American Chess Federation was not able or willing to satisfy.

With his admission to the Interzonal in jeopardy, Fischer started his preparation for the world series.

He began with the participation in the "USSR against the Rest of the World Match" of 1970. It was played by two teams of ten, ordered according to strength, each pair contesting four games between them. The Soviet Union won narrowly 20:19. Fischer's was a valuable contribution to this important encounter: two wins and two draws against the ex-world champion Petrosyan. He then played very well in two tournaments and on the first board for the USA at the Siegen Olympiad. At Buenos Aires, the place of his failure 10 years earlier, he won with an astonishing margin of 3 ahead of the rest of the field. At Siegen, Bobby's record was slightly marred by his game with Spassky. In an equal position, trying to win against the reigning world champion, Fischer took great risks and lost.

The invitation to the 1970 Interzonal in Palma de Mallorca was sent not to Fischer but to fellow American P. Ch. Benko (1928 –), who gracefully resigned in Fischer's favor. In a very strong field, Fischer again galloped ahead of everybody, with a 3 points margin. His triumph was complete: he achieved 3: against the Soviet contingent.

Finally, Fischer could play elimination matches with candidates for world championship. His quarterfinal and semi-final opponents were the Russian Taimanov and the Dane Larsen. The result was astonishing: Fischer won both matches with the clean score of 6:0, without losses or draws. Such dominance never occurred in the past and will probably never happen in the future.

In the final elimination match, Fischer met Tigran Petrosyan, winner of the other semi-final against Korchnoi with the score +1-0=9. The joke went around that it was Korchnoi who won. Indeed, nobody was eager to confront Fischer after what had happened. In the first game of the final, Petrosyan played the refutation of one of Fischer's favorite lines in Sicilian. It would have been easy for Bobby to avoid this way of play, but he was self-confident and stubborn. If somebody had found an error in his chess, he wanted it to be shown to him regardless of the consequences. This position won for Petrosyan, but he did not know what to do with it. Letting Fischer escape, he even lost the party. In the second game, in a quite different opening, Fischer played badly and was mated. He never explained the reasons of his weak play, so we don't know what happened in the next three games: was it depression or did he have a cold? He was lucky to save the games as draws. A "moral victory" for Petrosyan was proclaimed. After this, something strange happened. Following the openings, Fischer always reached positions where Petrosyan could hope only for a draw. Playing relentlessly, Fischer won the last four games one after another, ending the match with an excellent score +5=3-1, or 6:2. In these three matches and in the Palma de Mallorca tournament, Fischer had weak and strong stretches; however, nobody was able to profit from his weakness or to withstand his accelerations. The match with Spassky in Reykjavik, Iceland, was supposed to start July 2, 1972, but Bobby was still in New York. Perhaps he was unsure of the potential outcome of the match. Elo numbers, measuring chess player's strength, had been introduced by FIDE two years previously, and Bobby had the highest Elo number in the world. His tournament record was superior to Spassky's, but he never vanquished the Russian in personal encounters, losing three times. Another reason for Fisher's hesitation might have been the small amount of prize money. Only after the British chess patron, Slater, increased it by a donation of 50,000 pounds and dared Fischer to play, did the latter take the final plunge by flying to Reykjavik. In the first game, Fischer made a trivial error known to each beginner, and lost. He complained about noisy distractions by television technicians. The situation was not improved to his liking and Fischer defaulted the second game by not arriving to play. Fischer's score of 0:2 seemed particularly bad at the beginning of the third game when he allowed the lines of his King side pawns to be broken. His friends were appalled. But Spassky did not find the best reply. Fischer's position improved and he won a nice game. The games 3-13 were Fischer's run of wins; he made +6=4-1. After this, he was satisfied with draws. In seeking this, he had to switch to another opening system in each game to avoid analysis by Spassky's large research team. Fischer did not know how to make "grandmaster draws" by solid dull play that provoked the opponent either to answer in kind or to take undue risks. Instead, Fischer took risks himself. This, and stronger play by Spassky, explain the outcome of the second half +1=7-0, after many interesting games. Fischer became world champion. But after his triumph he stopped playing chess; he dived into oblivion.

However, there was an aftermath of these events after all. Bobby always wanted to play a return match with Spassky. His wish was granted in 1992 (???) when a Yugoslav millionaire provided a prize purse of \$3 million for this purpose. Played at the tourist and casino haven of San Stefan on the Montenegro coast, the match ended with Fischer's victory +10=15-5. "God has descended to Earth," commented Kasparov, implying that in the interval of 20 years Fischer's game had lost its magical properties. But by playing this

match Fischer, as usual, was placing himself in new difficulties. The match was in conflict with the American sanctions of that time against Yugoslavia. Fischer ended up as an expatriate in Europe. A return to the United States would have exposed him to financial or even criminal charges. If there is an expatriate who needs a pardon by a departing American president, it is surely Fischer.

History knows cases when a leading personality would die after achieving all the cherished dreams, reaching the summit of great efforts when each further step would lead downward. The end could as well be a dive into anonymity, into silence. The Russian poet Gumilev tells that after discovering America, Columbus became useless, "an empty shell." What could he have done comparable to this achievement? He selected humiliation, chains."

A new, powerful contender to world championship established himself in the 1970s in the Soviet Union, the young Anatoli Karpov (1951 –). The Soviet Chess Federation was prepared to put all its weight behind Karpov, deeming him superior to older Russian players. Karpov responded with enthusiasm to the request to submit evaluations of Fischer's weak and strong points [3]. In his match, Karpov was sure to have a team analyzing Fischer's openings. To be successful, Fischer, who was used to working alone, would have to change his style and rely on similar help. Nobody could have disapproved if Fischer had said, "This is what I have achieved single-handedly, fighting the Russians with all the power I had. Let somebody show now whether he can duplicate or even approximate my success." The outcome was that Fischer refused to play the match.

Who was Fischer?

In his formative years, between 12 and 14, Fischer played in many American chess tournaments open to players of widely varying strengths. Supported by his mentor, the chess master John W. Collins, Fischer managed to absorb the best features of these tournaments, avoiding their less desirable characteristics. Success depended here on winning most games against weaker players; this explains the aggressiveness of his style. However, Fischer learned to avoid a superficial, unsound way of playing. He acquired a clear, classical style, like Capablanca, but with much more acceptance of risk. Also Fischer's repertoire of openings was developed essentially at this time. Initially it was relatively narrow but, with time, became substantial and very deeply developed. With White, it was based on King's pawn step, e2-e4 at the first move. For Black, Fischer favored unsymmetrical, aggressive lines, Sicilian defense against e2-e4, Indian defenses against Queen's side openings, d2-d4. All of them gave both parties the possibility of vigorous initiative, perhaps with attacks on different wings of the board. The idea to obtain with Black initiative before equalizing was not new, but Fischer employed it all the time. Fischer mastered some openings, for example the Spanish for White so well that players avoided using them against him. He exhibited stubbornness and inflexibility in his choice of openings. For a long time he had predilections for the Sicilian "poisoned pawn variation." People joked, "It is bad even it is good (correct)." In this line Black would snatch an unprotected pawn and then defend a very complicated and unpleasant position with labyrinthine variations that were reassessed from one tournament to another. Relying on his own research, Fischer had success with it until a catastrophe against Spassky in 1972.

Fischer remained a positional player, but his games often contained a series of unexpected, tactical moves. One of Fischer's strengths was in his endgames, plays with a few pieces. He despised draws and would play as long as there remained a tiny bit of life in the position. Against top grandmasters he won some theoretically drawn positions by simply playing them perfectly and in a way least comfortable for the opponent. Fischer's play was remarkably free of oversights and he did not forgive any by his adversaries. He played best in positions where he had an advantage, however small, and was deadly if the opponent had no clear line of development. His play remained classically logical even in complicated positions; unpredictable, irrational sacrifices were absolutely alien to him. But Fischer played less well in complicated, chaotic positions with chances on both sides or when the position had changed radically. If Fischer lost it was not because of an oversight, but because of a bad position. Then he often lost horribly.

Fischer disliked dull play and never learned the art of "grandmaster draws." In this way of play, the player abandons his wish to win, having in mind only safety and balance of the position. The opponent then has the choice to reply in kind or to accept undue risks. This method of tormenting the opponent was used even in a world championship match. The contender to the title, the theoretically weaker Kasparov, used it in the match with Karpov in 1985. The overlong match of 48 games began with a score +5-0 for Karpov and was discontinued at +5-3, when the champion could not win a single one of the last 21 games. Its continuation

late in 1985 was won handsomely by the young Kasparov who learned a lot from his opponent. Twice Fischer found himself in a situation where draws would help him. In his mini-match with Petrosyan (1970) and the match with Spassky (1971) he was far ahead and tired. Rather than playing for draws, he took risks, merely moderating his aggressiveness. In the ten remaining games with Spassky he had nine interesting draws and one win. Fischer had a wonderful "sight of the board," quick and precise. This gift is especially useful for blitz-games, where the usual allotted time is 5 minutes per player and game. Tal was one of the famous blitz players. An enthusiastic blitz player in private, Fischer found blitz tournaments not serious enough. He participated in two of them, establishing his superiority over his contemporaries also in this respect. Just after the "Match of the Century" in 1970 a blitz-tournament was arranged in Yugoslavia. The best of the world took part. Fischer was first with 19 points out of 23 games, followed by Tal 14, Korchnoi 14, Petrosyan 13, Smyslov 9, Reshevsky 8. A little later, in the USA he played in a similar tournament for strong masters, achieving 23 of possible 24. In general, Fischer shunned also simultaneous displays when a master plays against several, perhaps 40 opponents. They were used even by world champions as an important source of income.

Modern tournaments are played with a chess clock that registers time spent by each player. Being in time-trouble, forced to play many moves in a short remaining time is a serious predicament, never experienced by Bobby. On the contrary, he usually had spare time at the end of the game. This explains his custom to come late to a play, forfeiting some thinking time. Was it arrogance or even an attempt to intimidate? Otherwise, Fischer was always praised for his impeccable, gentlemanly behavior at the board. Never would he try to explain or excuse his defeats. When playing at chess Olympiads, he was always ready to help his comrades. Indeed, Fischer's play hypnotized and intimidated his adversaries. After the interrupted (even-score) match with young Fischer in 1961, Reshevsky lost heart against him. As if hypnotized, he continued loosing to Fischer, due to either the latter's superior strategy or by his own strange errors. Other grandmasters had similar complaints. In 1970 the fifth game of the quarter-final match Fischer-Taimanov was adjourned in a drawn position, but with Taimanov's initiative. After analyzing throughout the night, Taimanov and his team decided to try for a win. Next day, after just a couple of moves, Taimanov captured with a rook an unprotected pawn by Fischer. When Fischer's queen gave a check attacking also the rook, the game was decided. In his game in Palma de Mallorca, Gligoric overlooked a fork by Fischer's knight. "We all play like idiots against Fischer," was the grandmaster's reaction.

It was against the organizers of chess events that Fischer was unforgiving and adamant. His complaints about lighting, noise by spectators, about disturbing photographers and television crews were exaggerated. Before the Lugano Chess Olympiad of 1968 where for the first time Reshevsky agreed to play on the second board after him, Fischer visited the premises, found the phosphorescent light unsatisfactory and left at once. Eventually his demands helped to establish acceptable standards for chess competitions. Financial issues about the prizes at tournaments were even more important. The Soviet Union claimed amateur status for its players, disregarding the fact that the best of them were compensated by grandmaster stipends, good apartments, travel expenses and help by chess experts. Fighting Soviet accusations of commercialism, Fischer replied that prizes at competitions were his only source of income and that a chess world champion deserves an adequate income. Single-handedly, Fischer affected a change in the prize from the dismal low of \$3,000 for the match with Taimanov to about \$100,000 in Reykjavik. This acceleration continued into the Karpov-Kasparov era with a budget of \$1,000,000 or more for a world championship match. The Soviet Union's unwillingness to pay these amounts forced the use of Western venues on the most important competitions and more Western influence to the sport that was dominated by Soviet players. Thus Fischer's protection of his own interests was a benefit for the game. The editor of the journal New in Chess, J. Timan, puts it this way: "Fischer had an unrestrained and open minded and at the same time a very professional view of the chess world, a very attractive combination."

Membership in USA chess societies jumped up during the Fischer era. He was popular also abroad, especially in Yugoslavia and countries of South America. In the USSR he was cherished by the intelligentsia, to which the larger part of the Soviet chess public belonged. This was their way to express a safe protest against Soviet reality or the media's endless claims of the superiority of Soviet players. When a translation of Fischer's "My 60 Memorable Games" appeared in the Soviet Union, one hundred thousand copies of the book were sold during the first three days [4]. Soviet grandmasters openly admired Fischer's games. After the Fischer-Taimanov match, they prepared an analysis of Fischer's play to be used against him. Confidential

at that time, it was published later [6, pp. 234-288]. This analysis reads like a panegyric for Fischer.

Now, 40 years after Fischer's retirement, he is not forgotten. On the contrary, during eager discussions about the best chess player of all times only two candidates emerged: Fischer and the present-day Kasparov. At the end, who won, Fischer or the USSR? The answer can only be: the latter. Fischer's victory was temporary. In spite or because of him the country, in its abundance of talent and dedication to the noble game, continued to produce flocks of excellent players.

The years 1958–59 defined Fischer as a man. He became grandmaster and candidate for world championship, an honor that many world-class players, like Shtein, were not able to achieve all their life. His friend J. Collins could not follow him to these heights with his advice and Fischer did not acquire new chess friends. In the Soviet Union he would have been assigned a trainer of good grandmaster strength. Fischer's entry into professional life could not have a worse beginning. It started with an interview he granted to the editor of a journal of doubtful reputation. The result was an article portraying Fischer in the worst possible light. The Soviet press, influential in the chess world, helped to complete a negative picture with its half-truths. Scornful of everything outside himself and his chess, Fischer understood little of the world he scorned. Regarded as anti-social and resentful of all authority, he became increasingly alienated from people [3]. He regarded himself as a defender of the purity and dignity of chess. He believed that very soon he would establish himself as the best chess player ever in the world. With age, another troubling aspect of his personality emerged. Not knowing the world and wrongly estimating its possibilities, he often suffered defeat in his most important endeavors. As if he wanted to be hurt! The following story he told of the 1980s in Los Angeles is a good illustration.

On one of his walks Fischer passed a bank, the scene of a robbery. The holdup man had escaped. The police just arrived and searched for the suspect. The passersby were questioned. Asked to identify himself, Fischer replied that he had no documents with him. "But you surely have your driver's license?" "No," replied he, "I do not have one, I never learned to drive." He also told them that he was a chess player. For the police this was hard to believe of a fourtyish American and they called their superiors. But Fischer refused to answer some of their questions. "I will not answer any question twice." So he was forced to spend a night in jail. Fischer described this incident in a little brochure published at his own expense. What kind of person is revealed by all this? Who was Fischer in the years of his chess activity? A Superman, aka Clark Kent of American comic strips? An "Übermensch," like Nietzsche's prophet–teacher Zaratustra? Our times are not favorable to superheroes of intellect. In democracies, hero–politicians are outweighed by the equalizing forces of society. Then there are monstrous semi–heroes like Stalin, Hitler, bin Laden. They could force even originally innocent people to participate in their crimes, hurt other people. But Fischer has hurt mainly himself. He remains a white knight of chess, preserving purity, logic and beauty of this most individual of intellectual activities.

After Fischer

The Soviet government's first reaction to Fischer's victory in Reykjavik was blaming all Soviet grand-masters for a "national disaster" because they were unable to retain for the USSR "our world championship." They faced harsher and harsher demands. Taimanov was the first victim. On his return, an unusually thorough examination by customs produced a forbidden volume of Solzhenitsyn and some foreign currency he failed to register. He found himself attacked by all party quarters, from his own small party cell to the Central Party Committee. Leading party ideologues could not understand his "clean" 0:6 loss to Fischer unless it was "a premeditated act in support of USA imperialism" [6, p. 218]. Taimanov was not allowed to play in tournaments abroad. He was deprived of his grandmaster stipend and the "Honorary Master of Sport" title and excluded from the Soviet Olympic team; in other words, his means of existence were thoroughly undermined. Reprieve began when the Dane Larsen, a grandmaster ranking above Taimanov, lost to Fischer with the same clean score. He could not be suspected to be an agent of American imperialism!

Significantly, Petrosyan did not suffer at all, but Spassky was made a scapegoat although he played much better against Fischer than Taimanov and Petrosyan. He was accused of laziness, of playing tennis instead of practicing the game for the match. Worst of all, he was blamed for disregarding the Sports Committee's order to claim the victory that became legally his when Fischer failed to appear at the second game of the match. Spassky "selfishly" continued to play. The 1973 championship was made exceptionally difficult for Soviet grandmasters. All of them except Botwinnik had to play. By a superhuman effort Spassky was able to win it, leaving, in this order, behind Karpov, Korchnoi, Petrosyan, Polugaevskii, Geller, Keres, Taimanov,

Tal and Smyslov. This was not enough: Spassky remained an undesirable person in his own country. The government wanted him to leave. When he married a staff member of the French embassy in Moscow the way for his emigration was opened and he settled in Paris.

During 1974, the young star Karpov won elimination matches against Polygaevskii, Spassky and Korchnoi. The last match, won narrowly with the score +3-2=19, was in effect a world championship match, for Karpov was awarded the title when Fischer declined to play. At that time the top four world players had the following Elo numbers: Fischer 2780; Karpov 2700; Korchnoi 2670; and Spassky 2650. As an indirect result of Fischer's win, the Soviet Union lost both its number 2 and 3 players, Korchnoi and Spassky. Indeed, in 1976, completing a tournament in the Netherlands, Korchnoi applied for and was granted asylum, claiming professional (not political) persecution in his homeland. According to Soviet laws, this was a criminal act. He was condemned in an open letter signed by most Soviet grandmasters. As usual with Soviet defectors, he became persona non grata. The USSR would boycott tournaments in which he participated. His name was never mentioned in the Soviet press, not even during his world championship match with Karpov in 1978 which the latter won again very narrowly with +6-5=21.

Korchnoi was known as one of the most tenacious, aggressive and tough chess players. Perhaps the fact that as a child he survived the horrible blockade of his native Leningrad explains this to some extent. Korchnoi was very different from Spassky, also a child of Leningrad. Six years younger than Korchnoi and evacuated to the Urals during WWII, Spassky developed a gentle nature. In Korchnoi's case we can observe the "immigrant effect." Some immigrants who had begun their successful professional career in the old country reached their peak abroad. Perhaps some obstructions of the old country disappeared and a surge of energy required to start a new life stimulated them. Korchnoi had his best results after he left the Soviet Union. During 1975–1980 he was the second after the dominating Karpov. He explained his defection by the complete absence of Soviet support for the elimination match against Karpov in 1974.

The period 1965–2000 can be called the time of great chess champions: Fischer in 1965–75; Karpov in 1975–85; and Kasparov in 1985–2000, when he lost to Kramnik, another Russian. Each of these periods saw a complete dominance of its champion. The Soviet Union and later Russia remained the dominant chess country.

Grandmaster Computer

This title is justified by the extreme importance of computers for chess players. It is only a matter of a few years before a chess-playing computer may be a world champion, superior to the best humans. Moreover, computers have become irreplaceable for everyday analysis, information storage and communications.

Many people identify computers with mathematics, but this is incorrect. Computers represent a technology, mathematics is a science. Computers produce numerical answers to many questions and disseminate and receive rapid messages. They depend on electronic chips that quickly perform simple additions. A programmer combines a great many of these simple additions into a mathematical expression that produces the required answer.

In contrast, mathematics describes properties of mathematical subjects by logically testable proofs. These subjects have become very complicated, for instance they can be functions instead of just numbers and spaces of infinitely many dimensions instead of just the Euclidean plane. Mathematics proved to be astonishingly helpless to teach good chess playing. On the one hand chess is trivial, deterministic. In each position there are only three possible outcomes: win, loss or draw for White; there are no secrets. On the other hand chess is almost infinite, inexhaustible in its abundance of possibilities. True, there is a branch of mathematics called Game Theory, in which J. F. Nash (1928 –) excelled. But it is a theory for games that are shorter and indeterministic, where a player answers an opponent's move without actually knowing the move or the position completely.

It has been known for a long time that computers may be programmed to play chess. There are several ways for this. The simplest one uses "the brute force" of the computer. To choose a move, a player examines the position this way: "If I play this, then he will answer that; if I play another move, then.." In this process, the player is able to examine only a few main possibilities. However, the computer can examine all possible moves, and there are many. As an example, we take the position in the game Capablanca–Bernstein, St. Petersburg, 1914, after the 16th move. In this position there are 43 possibilities by White and 38 by Black—that is, more than 1000 double moves. We shall take this number as a typical number of moves in a chess position. It has 1000 first moves, then 10002^{106} sequences of two moves, and 10005^{1015} or one

thousand trillion sequences of five moves, which is already outside of a computer's range. A club player often makes two move blunders, a grandmaster only rarely, but he often cannot see as far as five moves. Here the computer has a clear advantage. Another way of programming may be by installing positional commands. The computer should be required to gain material, if possible, to avoid positions with weak points or with an unprotected king, to strive to open lines for the rooks, and so on. This the computer cannot do as well as humans. The third approach stores catalogs in computer memory: variants of recommended openings, plays of standard end games. All these methods should be adjusted to work together. In particular, the computer should be able to estimate positions produced by the first method.

Chess playing computers appeared for sale around 1966. Their strength improved and ten years later only few chess players could beat them. Thus, companies lost commercial interest in further improving computers, except for the prestige that could arise from a victory over a famous player. This really happened in 1997, when IBM's Deep Blue won a match against the world champion, G. Kasparov. A contest of V. Kramnik against the Deep Fritz of Einstein TV ended in 2002 in a draw, +2-2=4. In 2003, there was another draw by Kasparov, +3-3=0.

A chess computer has certain non-human properties in its favor. It can "think" faster, has no time troubles, cannot be intimidated and does not give up the game until mated. It sees through all swindles. A grandmaster compared playing against a computer to sitting opposite a black hole. He can play positionally somewhat better, but one mistake and the game is gone. The player does not know whether the computer estimates its position to be better or worse. Even if one would win against a computer, it is expected that a slight improvement of the computer will make it unbeatable. But what does all this matter? After all, we don't ask a marathon athlete to compete against a bicyclist.

A human grandmaster of our time cannot travel without his ally and helper-his personal computer. It contains variations of openings that the player wants to use. He gets online all games ever played and recorded with these variations-also by mediocre players who sometimes can have the idea of a genius. He can analyze games played by himself and by his opponents. The problem of some simple end games such as queen with a knight or a bishop against queen has been completely analyzed and solved, that is, it has been decided whether they are as a rule won by the stronger side or not.

Two Problems

I would like to propose two chess-mathematical problems, the answers to which we can only guess. The first one is the Main Problem of Chess, namely: Is the initial chess position won by White? White has an obvious advantage because it has the first move. Is this position drawn or is it won by White? In other words, can White, by playing best moves, win against any defense of Black? If not, there must be a strategy that allows Black to defend itself in all variations.

In any large set of chess games, independently selected, White has a marked advantage. For example: Fischer, in his eight USA championships achieved +35-2=9 or 86for the world championship in 1970–71 he had +18-1=12 with White, +20-3=9 with Black—that is, 77

If we want to apply computers to answer our question, we should use the "brute force" method; the positional evaluations are not sufficiently reliable. It is important to know that the length of a chess party is bounded by some large number. The reason is the 50 move rule of chess: a game is drawn if during any 50 successive moves no pawn move and no piece capture have occurred. There are 30 pieces to capture; a pawn has at most nine possible moves (including promotions to different pieces). This gives at most 30+9x16+1=175 periods of 50 moves and at most 175x50=8750 moves in a chess party. With 100 possibilities at each move (much less than in the example above) we get 1008750=1017500 chess games. This number is ridiculously large. Even if we take a more realistic estimate of 40 moves per game, we still get 1080 games. No computer can check that many possibilities. The answer to our question is that it cannot be answered exactly at the present time. What is still possible is a statistical approach. Our understanding of chess openings and of the general strength of play has increased with time. Statistically, is this more favorable for White or Black? In the same way one can compare percentages of success of stronger and weaker players.

My second question is: Are the abilities for chess and for mathematics (and other exact sciences) similar or essentially different? I will try to prove that they are different. Different opinions abound. The mathematician G. H. Hardy joked that mathematics is deeper than chess. Indeed, in chess you can sacrifice at most a queen, while in a mathematical proof by contradiction, one sacrifices the whole game by assuming that the statement to be proved is wrong and shows that this leads to absurdity. On the other hand, chess is

more addictive than mathematics. A loss in chess—with symbol 0—is a bitter defeat, while in mathematics, a failure to prove a theorem is just a draw, scoring 1/2. One can come back to the problem with fresh ideas later in a new attempt to solve it. Chess is only a game, a sport that tries to solve the mystery of the initial chess position. The location and moves of pieces in it are arbitrarily fixed by tradition, not derived by logic or some law of nature. In contrast mathematics, as it comes alive in a good research paper, is an investigation of a part of reality or of an important creation of our mind, derived from the inner logic of our science. This tends to imply that they are different.

I think that the kernel of the problem lies in the different natures of chess and of exact sciences. According to the philosopher K. R. Popper, an exact science is a logical system whose statements are open to everybody to test them, with the purpose to prove them false. Mathematics is one of these sciences. To explain this, I will describe the work of a research mathematician. First, in his field of study he has to find a worthwhile problem. We can assume that he wants to prove a new theorem that he hopes is true. He has then to find a sketch of its proof—this is the most difficult and creative part of his work. This consists of key statements to be used for the proof. Adding details, he obtains a chain of statements, each of them following from the preceding ones and known facts and ending with the statement of the theorem. All this can be done in his office, at home, in one day, in a month or in two years. All the time he has a definite goal before him, to prove his theorem. It follows that mathematics is an exact science in Popper's sense: a critic can test the logical validity of the steps of the proof and the correctness of the facts used, including the axioms of this field of mathematics.

A chess master "works" in a different way. In high quality tournaments only one game is played per day, with perhaps five hours for the first 40 moves, then two additional hours for the remainder of the game. Some moves are obvious, require no thinking time, but in difficult positions a plan ahead as far as possible is required. A player selects possible candidates for the first move, then plays through in his mind each of them in turn, considering his own and opponent's moves. Clearly, errors occur. Often even a great master does not know whether his move, plan of play or estimation of position was correct or "false." This applies not only to his game on the board but also to later analysis with the help of a computer. At some, perhaps the most important moments of the game, the player relies not only on logic but also on his instinct, his positional feeling. The work of a mathematician can be checked, and he has freedom in his selection of the subject. The opposite is true of a chess player. He has no protection from error in positions that may have been dictated by his opponent.

One can approach our question from another side. How many players of grandmaster strength were also creative in mathematics? Several were teachers of this subject. Euwe taught mathematics and computers in college. Botwinnik and Spassky had a short affair with mathematics during the first year of their study at St. Petersburg University but gave up mathematics as too demanding and interfering with a chess career. The German-Jewish world champion Emanuel Lasker (1862-1941) was a person of many cultural interests. He kept his title for a record 28 years. A player of enormous practical strength, Lasker did not care about beauty in his games but liked the tactical and psychological struggle in difficult positions. An outflow of his ideas about chess enriched his philosophical books about life, like his "Philosophy of the Uncompletable" of 1919. Much more influential were Lasker's results of his mathematical doctor dissertation at the University of Erlangen in 1900. They became an important component of a new form of algebra that originated in Göttingen in Germany and had an explosive effect. Lasker's useful lemmas appear everywhere in presentday graduate textbooks. During his short stay in Moscow in 1935-37, he has been elected to an honorary membership in mathematics at the Academy of Sciences. This honor was reserved to developers of schools of scientific research, like mathematicians Lusin and Grave. This is the only example that I know and that I remember that there is no rule without exception. We can conclude that mathematics and chess are very different intellectual activities and that they require very different abilities.

Grandmaster Tartakover contends: "I make mistakes, therefore I am [a chess player]." This could be opposed by: "My mistakes can be discovered by checking, therefore I am a mathematician." The lure of chess for youth in the Soviet Union did not deprive Stalin of many good mathematicians or atomic physicists. It is more probable that, as they grew to grandmasters, the bloodless aggressiveness of chess served for these youths as a sublimation of a more dangerous involvement in politics. As an eighteen year old in the Soviet Union, choosing mathematics in preference to engineering, I was aware of these thoughts.

Bibliography

- [1] M. M. Botwinnik, Analiticheskie I Kriticheskie Roboty, vol. 1-4, 1984-1987, Moscow.
- [2] R. J. Fischer, My 60 Memorable Games, Simon and Schuster, 1969, New York.
- [3] D. Hooper and K. Whyld, The Oxford Companion to Chess, 1984, Oxford Univ. Press, Oxford.
- [4] V. Korchnoi, My best games, Olms, 2003, Zrich.
- [5] L. S. Pontryuagin, Zhizneopisanie, Sostavlennoe im Samim, I Ch. P "Pima B", 1998, Moscow.
- [6] Russians versus Fischer, D. Plisetsky and S. Voronkov, eds., 1994, Chess Books, Moscow.
- [7] Shakhmaty, Enziklopedicheskii Slvoar, [Dictionary], A. E. Karpov, ed., 1990, Sovetskaya Enziklopediya, Moscow.
 - [8] G. Sosonko, Russian Silhouettes, 2001, New in Chess, Alkmaav, the Netherlands.