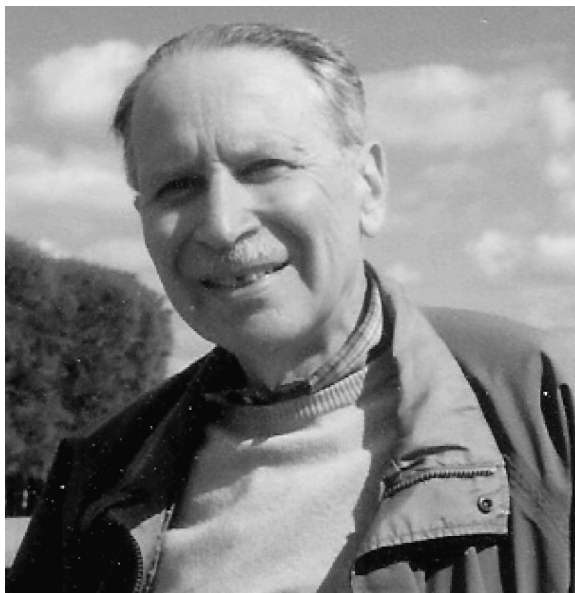


In Memoriam

Theodore J. Rivlin (1926–2006)<sup>☆</sup>



Theodore J. Rivlin, our friend and mentor

It is, with heavy hearts, that we report that our friend and mentor, Theodore J. Rivlin, died on July 22, 2006, in Croton-on-Hudson, New York, just weeks before his 80th birthday. He had suffered from Alzheimer's disease in his final years. Ted was an associate editor of *Journal of Approximation Theory* from its inception in 1968.

Ted was born on September 11, 1926, in Brooklyn, New York. He received his undergraduate degree at Brooklyn College in 1948. After serving for 18 months with the U.S. Army Air Force, he continued his mathematical education at Harvard University, where he received his Ph.D. in 1953 under the supervision of Joseph L. Walsh. His thesis was concerned with the phenomenon of overconvergence of power series.

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<sup>☆</sup> This obituary is based on an article by the first of us, CAM, published in *Ann. Numer. Math.* 4(1) (1997), viii–xi.

He began his first position in 1952 as an instructor at Johns Hopkins University in Baltimore. In 1955, he left Baltimore to become a research associate at the Institute for Mathematical Sciences at New York University (now called the Courant Institute of Mathematical Sciences). A year later, he chose to accept a position in industry as a senior mathematical analyst at the Fairchild Engine and Airplane Company in Deer Park, Long Island. It was during this period that his work on developing tables for thermodynamic functions led him to approximation theory and Chebyshev polynomials.

In 1959, he decided to join the recently established T.J. Watson Research Center. Except for two sabbatical leaves, one from 1969 to 1970 at the Computer Science Department of Stanford University and the other during the years 1976 to 1977 at the Mathematics Department of the Imperial College, London, he served the IBM Corporation as a member of their research staff for nearly 35 years until he retired as an emeritus staff member.

Ted Rivlin also held an adjunct professorship of mathematics at the Graduate Center of the City University of New York from 1966 to 1976, where he lectured on approximation theory. He continued to serve as an associate editor of the *Journal of Approximation Theory* as well.

Ted was the author of over 80 research articles in mathematics which span a wide variety of topics in approximation theory and computational mathematics. He wrote three books, one on approximation theory, titled *An Introduction to the Approximation of Functions*, appeared in 1969 and was reprinted in 1981 (see [B1]), and the other two were on Chebyshev Polynomials. The first one of these, titled *The Chebyshev Polynomial*, appeared in 1974 (see [B2]), and the other, called *Chebyshev Polynomials from Approximation Theory to Algebra and Number Theory*, was published in 1990 (see [B4]).

His three books are masterpieces of mathematical exposition. The one on approximation theory quickly became a standard text. It has influenced a whole generation of approximators and is still a widely consulted source of information on approximation theory. Likewise, his books on Chebyshev polynomials are important reference sources and continue to influence the development of this subject.

Ted's ability as an expositor of mathematical ideas ranks high among his talents. His mathematical prose always succeeds to clarify even the most technical concepts and results. However, it is the novelty, depth, and beauty of his mathematical discoveries that we admire and appreciate the most.

On the dust jacket of his 1974 book *The Chebyshev Polynomials* is a picture<sup>1</sup> of Ted posing next to a stone bust of P.L. Chebyshev located on the grounds of Moscow State University. This picture, taken during the International Congress of Mathematicians held in Moscow in 1966, more than any other image or words reflects Ted's mathematical interests. Chebyshev was the founder of the theory and practice of approximation. Ted had contributed widely to this subject and had significantly influenced its development during the past 40 years.

In one of his earliest works on best approximation in the maximum norm written with H.S. Shapiro [77], the useful notion of extremal signature was introduced. This paper, as well as others that he wrote on best approximation, led to numerous subsequent developments. Noteworthy in this regard is his work with Shapiro [76] on strong uniqueness, with B.R. Kripke [22] on best approximation in the  $L^1$ -norm, and with R.J. Sibner [78] on best approximation of bivariate functions. In [72], with E.W. Cheney, the connections of best approximation on an interval to best approximation on a finite subset of it are studied with the idea of practical computational value.

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<sup>1</sup> [http://math.nevai.us/AT/RIVLIN/PHOTOS/rivlin\\_cheb.jpg](http://math.nevai.us/AT/RIVLIN/PHOTOS/rivlin_cheb.jpg).

His paper [35], with D.J. Newman, provides sharp estimates for the approximation in the maximum norm of a monomial by lower degree polynomials. Such problems about “incomplete” polynomials have been an active area of research in approximation theory, and this paper was one of the earliest contributions to this subject. The exact form of best polynomial approximation in the maximum norm to a given function is rare. The Chebyshev polynomials are an example. Ted pursued this issue in his papers in [47,50,80] and also through his lifelong interest in Chebyshev polynomials.

Ted looked at Chebyshev polynomials from many angles. In his paper [1], with R.L. Adler, he treated their ergodic properties. In another direction, he studied with Luttmann [23] the Lebesgue constant for polynomial interpolation at the zeros of the Chebyshev polynomial. This work provided both computational observations and an asymptotic analysis of the Lebesgue constant at the zeros of the Chebyshev polynomials. He returned to these issues many times. In [81], he discovered that the maximum norm error in expanding certain functions in a series of a large class of Jacobi polynomials is least for the truncated Chebyshev expansion. C. Lanczos introduced an important technique for constructing good approximations to certain functions which is fundamentally based on Chebyshev polynomials. It is no surprise that Ted was attracted to this practically useful approximation technique, which he expounded upon in a series of three papers [79,18,51].

Ted wrote several papers on Bernstein polynomials. In [17], with R.P. Kelisky, he considered the limit of the powers of the  $n$ th Bernstein polynomials where the power chosen depends on  $n$ . This novel idea had unexpected applications to the identification of the saturation class of the Bernstein polynomials. When their results are combined with techniques developed by P. Butzer of RWTH, Aachen, for the study of saturation of semigroups, the saturation class of Bernstein polynomials (which do not form a semigroup) can be identified. Their paper continues to influence the study of approximation properties of positive operators and also provided the beginning of the Ph.D. thesis of the first of us, CAM, written under the guidance of S. Karlin at Stanford University. Ted’s other paper on Bernstein polynomials [53] is an early work on the use of what is now called “degree raising”, which is a useful concept from geometric modeling. A polynomial  $p$  of degree  $n$  expressed in the Bernstein basis can be rewritten in terms of Bernstein bases of any degree higher than  $n$ . The coefficients of these higher degree representations of the given polynomial will converge to  $p$  and therefore can be used to estimate the maximum in absolute value of the polynomial.

As is often the case, mathematical progress is slow, unpredictable, and fraught with frequent setbacks. One of Ted’s papers with D.J. Newman [34] stems from their days as Ph.D. students at Harvard University. At that time, the second of us, RSV, was also a graduate student at Harvard. Ted and RSV both were in residence at Pierce Hall, and on one occasion Ted presented RSV with a conjecture about the zeros of the partial sums of the Maclaurin expansion of  $\exp z$ . After computing the zeros of the first three polynomials, Ted conjectured that the zeros of all the partial sums lay in the left half plane. RSV was fascinated by the conjecture and quickly showed that a horizontal strip, of semi-width  $\sqrt{6}$  about the positive real axis, was free of zeros of all partial sums, a far weaker result than Ted had conjectured. But a few days later, Ted learned from Professor Walsh that G. Szegő, in 1924, has shown that these zeros can lie in the right half plane, and moreover that they have positive density there. RSV’s strip result was thus new, and it became his first published paper in 1952, and continuations of this work became a part of his thesis in 1954. It should also be mentioned that Ken Iverson, a graduate student at Harvard, who was later to become the inventor of APL (for A Programming Language), did more extensive numerical experimentation on these zeros on a Mark IV computer at Harvard.

There was a long hiatus and only in 1972 did Ted return to the problem [34]. In that paper, he and D.J. Newman established that there exists a symmetric parabolic shaped region about the positive axis free of zeros of all partial sums of  $\exp z$ . An error in [34] was found and an erratum was published in 1976. Subsequently, RSV and E.B. Saff showed for a large class of entire functions (including the exponential function as a special case) that there was a parabolic region encompassing the origin and symmetric about the horizontal axis free of zeros of all the partial sums of its Maclaurin expansion. For a complete exposition of these ideas, see the book by Albert Edrei, E.B. Saff, and R.S. Varga titled *Zeros of the Sections of Power Series*, Lecture Notes in Mathematics, Vol. 1002, Springer, Berlin/New York, 1983.

Anyone devoted to computational mathematics cannot escape the lure of numerical quadrature. Its intrinsic beauty and practicality draws us inexorably toward it. Ted succumbed to it several times. In [25], Turán's formula for the Chebyshev weight function was obtained using certain divided difference functionals at the zeros of the Chebyshev polynomials repeated with multiplicities. This formula led to a solution of one of Turán's open problems. Ted had other interests in Gaussian quadrature. In [27], quadrature formulae with a degree of accuracy within two of the Gaussian case were classified and, in [68], Ted's interests turned to error estimates for Gauss quadrature of analytic functions.

Divided differences are a useful tool in computational mathematics, and so Ted was naturally interested in questions that pertain to divided differences. In particular, one might wonder when the divided differences of a function on a triangular array of points determine the function. This surprisingly difficult question was studied in [40,16], while in [36], the difficult nonlinear problem of characterizing the weights of a divided difference functional was solved.

Applied mathematicians often approximate. Frequently they benefit from approximation theory's nearly inexhaustible supply of approximants. Sometimes it appears that the development of practical approximation techniques is more an "art" than a "science". Consider the typical situation of recovering a function from some of its values. Without further information, nothing can be said about its value at other points. Nonetheless, one can always choose some method to approximate it based on the available function values and then assess the error using some a priori information on the function, typically given in terms of some information on the regularity of the function. However, if error bounds are useful, then it may be worthwhile to find the best such bound amongst all estimators (even nonlinear ones) for the function based on its given function values. This problem is solved in [29], which began Ted's work on optimal recovery. His curiosity about optimal recovery led him to return to it more often than to any other subject.

A striking result of Walsh states the following: the difference between the  $n$ th Maclaurin polynomial of a function and its Lagrange interpolation at the  $n$ th roots of  $z$ , where  $|z| = r < 1$ , goes to zero exponentially fast as  $n \rightarrow \infty$  in the disc centered at the origin with radius  $r^{-1}$ , for any function analytic in the unit disc. Ted made two interesting contributions [63,33] to the understanding of the phenomenon of Walsh equiconvergence by relating it both to best approximation and to optimal recovery.

We mention that a very complete treatment of this phenomenon has recently appeared in *Walsh Equiconvergence of Complex Interpolating Polynomials*, by J. Szabados, A. Sharma, and A. Jakimovski, Springer-Verlag, 2006; see also *Selected Papers of J.L. Walsh*, edited by T.J. Rivlin and E.B. Saff, Springer-Verlag, 2000.

In summary, the work of Theodore J. Rivlin has been a major influence on the development of contemporary approximation theory. His lifelong commitment, enthusiasm, and devotion to the subject are an inspiration to all of us who work in this field. We have been fortunate to have

shared with him similar mathematical interests and to have him as a friend and mentor. For this gift, we are thankful.

Ted is survived by his wife, Jean, two daughters, Madeline and Elizabeth, and three granddaughters, Cynthia, Emily, and Alice. We will all miss him.

Charles A. Micchelli  
Mohegan Lake, New York

Richard S. Varga  
Kent, Ohio

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