End degrees and infinite cycles in locally finite graphs

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Abstract. The cycle space of a finite graph has a number of well-known properties. Diestel and Kühn recently proposed a notion of the cycle space which made possible to extend most of these properties to locally finite graphs. However, one of the most basic properties, namely the characterisation of the cycle space elements by vertex degrees, seems to fail. In this talk it will be shown how this can be rectified by introducing "end degrees". These end degrees are based on counting the number of edge-disjoint rays in an end. In addition to giving a characterisation of the cycle space, end degrees allow generalisations of the following simple facts for finite graphs:

- the number of odd vertices is always even;
- a subgraph is a cycle if and only if it is connected and 2-regular; and
- if the minimum degree is $\geq d$ then there is a cycle of length $\geq d + 1$.

The talk is based on joint work with Maya Stein.