# Nowhere-Zero 3-Flows and Odd $K_{4}$-Partitions 

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#### Abstract

An odd $K_{4}$ is a subdivision of $K_{4}$ such that all four cycles corresponding to triangles in the $K_{4}$ are of odd length. It was shown by Catlin that every graph containing no odd $K_{4}$ is 3 -colorable. The purpose of this paper is to establish the following dual version of Catlin's theorem: every 2-edge-connected multigraph with no odd $K_{4}$-partition admits a nowhere-zero 3-flow, where an odd $K_{4}$-partition of a multigraph $G=(V, E)$ is a partition $\left\{V_{1}, V_{2}, V_{3}, V_{4}\right\}$ of $V$ such that (i) $G\left[V_{i}\right]$ is connected for each $1 \leq i \leq 4$; (ii) there is at least one edge between $V_{i}$ and $V_{j}$ for each pair $1 \leq i<j \leq 4$; and (iii) the number of edges between $V_{i}$ and $V \backslash V_{i}$ is odd for each $1 \leq i \leq 4$. (Co-authored with Xujin Chen and Wenan Zang)


