Nowhere-Zero 3-Flows and Odd K₄-Partitions

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Abstract. An odd K_4 is a subdivision of K_4 such that all four cycles corresponding to triangles in the K_4 are of odd length. It was shown by Catlin that every graph containing no odd K_4 is 3-colorable. The purpose of this paper is to establish the following dual version of Catlin's theorem: every 2-edge-connected multigraph with no odd K_4 -partition admits a nowhere-zero 3-flow, where an odd K_4 -partition of a multigraph G = (V, E) is a partition $\{V_1, V_2, V_3, V_4\}$ of V such that (i) $G[V_i]$ is connected for each $1 \le i \le 4$; (ii) there is at least one edge between V_i and V_j for each pair $1 \le i < j \le 4$; and (iii) the number of edges between V_i and $V \setminus V_i$ is odd for each $1 \le i \le 4$. (Co-authored with Xujin Chen and Wenan Zang)