

YMC 2004

Abstracts of Student's Presentations

August 20-22, 2004

The Ohio State University

YMC2004 TALK ABSTRACT

Jason Allen Anema

Purdue Universtiy

Title: Riemann Zeta Function as a Conformal mapping

Description:

The Riemann map (Ahlfors map if the domain is n connected with $n > 1$) of a domain $\{z: \text{mod}(Zeta(z)) < p\}$ is an algebraic function of Zeta. Investigating this leads to a generalization of a domain of the form $\{z: \text{mod}(Q(z)) < 1\}$ for Q analytic, that the Riemann (Ahlfors) map f of such a domain is completely determined by a finite number of images of f , namely at the zeros of Q . Properties of the Bergman Kernel in connection with the Riemann (Ahlfors) maps are also described. It was known previously that the Bergman Kernel is algebraic iff the Riemann (Ahlfors) maps are algebraic. With some conditions put on the boundary of a domain it is found that the Bergman Kernel is an elementary function iff the Riemann (Ahlfors) maps are.

YMC2004 TALK ABSTRACT

Xiaoling Ang

Loyola University Chicago

Title: Explorations of the Henon Map for Small Values of a and c

Description:

The study of Hénon transformations is of particular importance because they are one of the simplest examples of maps with complicated dynamics. They are also of particular interest in applied mathematics (such as in establishing Pareto equilibria in economics) because any polynomial automorphism on the plane can be conjugated to the composition of generalized Hénon transformations (1). This project, conducted under Dr. Roberto Hasfura at the Trinity University 2003 REU, characterizes the dynamics of the Hénon map

$$f(x,y)=(y, y^2+ax+c) \text{ from } \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

for $0 < a < 1$, $c=0$, and seek to explore the parameter space of maps topologically conjugate to f by varying c . This study makes use of machinery developed previously in \mathbb{C}^n as well as classical results developed by Palis and Smale in the 1960s.

(1) Shmuel Friedland and John Milnor. "Dynamical Properties of Plane Polynomial Automorphisms." *Ergodic Theory and Dynamical Systems* (1989), 9, p. 67-99.

YMC2004 POSTER ABSTRACT

Julie Bjornstad, Alexei Dachevski

University: Georgia Institute of Technology

Title: Modeling of Phytoplankton Food Chains

Description:

The main focus of this project is modeling phytoplankton predator-prey systems involving a resource, prey (phytoplankton), and a predator (zooplankton) in order to understand the complex interactions between these elements. It is very important to study phytoplankton food chains because phytoplankton contribute to numerous biochemical processes. They play an important role in controlling the water quality and exert a great influence on the global climate by regulating the carbon dioxide uptake. Phytoplankton constitute the base for most aquatic food chains and are relatively easy to study because they are simple, small creatures with short life spans. In addition, lab and field experiments are feasible. Experiments in chemostats can be readily combined with theoretical analyses. Simpler cases involve a resource and one to two phytoplankton species. The more complex cases investigated consist of food chains of all three species. All models include fluctuations in resource availability and seasonal variations. Our approach is through equilibrium analysis, linear stability analysis, and numerical simulations of differential equations. We are interested in both the short term as well as long term behaviors. So far, results indicate that for long periods, stable dynamics exist and that for shorter periods, relaxed oscillations are present. Also, as the total nutrient content is varied, the system goes through several bifurcations, resulting in drastic changes in the overall system behavior ranging from stable coexistence of both species to dominance of just one to extinction of both.

YMC2004 TALK ABSTRACT

Grant Michael Boquet

Virginia Polytechnic Institute and State University

Title: GRiTS: Global Re-indexing for Triangular (Tetrahedral) Simplices

Description:

GRiTS is a project started in the Winter of 2003. I gave a talk on it in May to our math department - below are the slides I used.

Description of Research:

Finite Elements is one of the most popular approaches for finding numerical solutions of PDEs. For problems arising in complex engineering and scientific applications, the number of variables can easily reach millions or even billions. To reduce the associated computational time, one of the most popular approaches is parallel computation, in which the computational load is spread over hundreds of processors. Even with decreasing latency of networks, one major challenge is the data transmission. To distribute the amount of computational work evenly over processors (load balancing), ParMETIS, a hypergraph partitioning package, is used in conjunction with several mapping algorithms which were created. This research introduces various algorithms for mapping a hypergraph to a Finite Element mesh, which are able to reduce computational time by preventing stalling and reducing network communication. Analysis of each algorithm and experimental results demonstrate as much as a 35% speed up in computations.

Paper Abstract:

In this paper, we describe and analyze a number of index filters associated with graph partitioning. A heuristic filter based on two level adjacency and random sampling demonstrated the best performance. A numerical study using a parallel finite elements flow solver (ViTLES) and the parallel graph partitioning package ParMETIS is included.

The slides I used can be found at:

<http://filebox.vt.edu/~gboquet/grits.pdf>

I preliminary paper can be found at:

<http://filebox.vt.edu/~gboquet/paper.pdf>

YMC2004 TALK ABSTRACT

Samuel J Buelk

College of Charleston

Title: Two Block Size PBDs with a Maximum Number of Triples

Description:

The intent of the research is to find the necessary and sufficient conditions to construct a PBD $(v, \{3, k\}, 1)$ with $k = 4$ or 5 having a maximum number of triples.

YMC2004 POSTER ABSTRACT

Nate Burch

Grand Valley State University

Title: Least Squares Properties of the Lanczos\' Derivative and Higher Order Lanczos\' Derivatives

Description:

The Lanczos\' generalized derivative, developed by Cornelius Lanczos, is a proper extension of the normal derivative. We show that this derivative satisfies a least squares property.

We used this result to construct higher order Lanczos\' derivatives. Properties of Legendre polynomials and summability kernels play a prominent role in this construction.

YMC2004 POSTER ABSTRACT

Thomas Callaghan

Georgia Institute of Technology

Title: Random Walker (Monkey) Rankings for College Football

Description:

The Bowl Championship Series (BCS) National Championship for NCAA Division I-A football is currently determined by the outcome of a single bowl game played between the #1 and #2 teams in the BCS Standings, which includes both polls and ranking algorithms. The difficulty in selecting these two teams lies in the fact that each of the 117 Division I-A teams plays only approximately 11 regular season games. Using only the win-loss information of those games, we develop a family of rankings defined in terms of independent random walks on the graph of teams (vertices) and games played (edges), in one parameter (the bias of the walk). The virtue of this ranking system lies in the simplicity of its explanation. We compare results for recent seasons against BCS Standings and its components, demonstrating that the results from this simple algorithm is competitive with more complicated methods. We investigate the statistical properties of the randomly walking voters and examine the asymptotic properties of the rankings at extreme values of the single parameter. We also investigate the connection between the rankings and the underlying community structure of the network of games played.

YMC2004 TALK ABSTRACT

Kimberly Cervello

State University of New York at Geneseo

Title: Extraction Degrees of Zero Sequences in Finite Abelian Groups

Description:

At the Research Experience for Undergraduates Program at Trinity University, in San Antonio, Liya Zhu (U C Berkely), Denise Terry (University of Redlands ? California), and I investigated block monoids using Combinatorial/Algebra,. Specifically, we studied finite abelian groups and their zero sequences. The set of all zero sequences in a finite abelian group G , $ZS(G)$, is the set of all sequences, repetition allowed, order unimportant, whose sum is the zero element in G . The set of all minimal zero sequences in G , $MZS(G)$, is the set of all irreducible zero sequences.

Most importantly, we studied the extraction degrees of these zero sequences. For example, let $X=1,1,1,1$ and $Y=1,1,1,1,1,1,2,2,2$. Then, X and Y are zero sequences in Z_4 . Now, the extraction degree is defined as the supremum of β over α such that $(\beta)(X)$ is contained in $(\alpha)(Y)$. Using X and Y as defined above, we see that $(1)(X)$ is contained in AT LEAST $(1)(Y)$ and $(2)(X)$ is contained in AT LEAST $(2)(Y)$, and so on. Thus, the greatest fraction that produces this result is 1 and the extraction degree of this fixed X and fixed Y is 1.

We have discovered and proven theorems explaining what extraction degrees are possible and when they are possible for finite abelian cyclic groups. Furthermore, we developed various general theorems concerning the extraction degree, which make the extraction degree easier to find. Lastly, I am currently working on extending our results and producing new results concerning the extraction degree for finite abelian groups of rank 2 and higher. At the Ohio State Conference, I look forward to introducing this topic and presenting some preliminary results from our research.

YMC2004 TALK ABSTRACT

Adrienne Chisholm

College of Charleston

Title: Graph Theory/Non Regular Non Planar Graphs

Description:

This is an interdisciplinary project involving both Math and Computer Science. A program will be developed which, when given a finite sequence, will determine whether that sequence is graphical. If so, the program will draw a corresponding planar graph if possible. If the graph cannot be drawn as planar a nonplanar representation will be drawn. The aim of this project is to give arguments why a particular graphical sequence or family of graphical sequences has no planar representation.

YMC2004 TALK ABSTRACT

Kristen Dardia

James Madison University

Title: Using Scale Mixtures of Normals to Model Continuously Compounded Return

Description:

In this paper, a new method for estimating the parameters of scale mixtures of normals is introduced and evaluated. The new method is called UNMIX and is based on minimizing the weighted square distance between exact values of the density of the scale mixture and estimated values using kernel smoothing techniques over a prespecified grid of x-values and a grid of potential sigma values. Applications of the method are made in modeling the continuously compounded return of stock prices. Modeling this ratio with UNMIX proves promising in comparison with other existing techniques that use only one normal component, or those that use more than one component based on the EM algorithm as the method of estimation.

YMC2004 TALK ABSTRACT

Laura Dev and Sara Lapan

Tufts University, University of Chicago

Title: Modeling Cancer Mathematically

Description:

The purpose of this project is to create a mathematical model of tumor growth and angiogenesis. This model analyzes the effects of cell density and the concentrations of fibronectin, a protease enzyme, growth factor, and various inhibitors on the movement of endothelial cells along the capillary wall. We used enzyme kinetics, random walks, and systems of differential equations to derive mathematical relationships among the above 5 components. From these equations and the help of MATLAB, we were able to simulate the onset of tumors and prove that certain systems are inherently unstable, while others can be controlled by the presence of certain inhibiting factors.

YMC2004 TALK ABSTRACT

Anthony DiPietro

Grove City College, PA

Title: 3D Edge Detection and Visualization based on the Geometric Heat Equation

Description:

In recent years, the application of partial differential equations to image processing has emerged as an active area of research, and has led to several discoveries in fields such as medical imaging and computer vision. The goal of our research is to develop an edge detection and visualization method for images in 3D with a large capture range and having the capability of handling images of varying sizes. We consider a convex, closed surface and construct our model according to the geometric heat equation to shrink the initial surface around the object contained in the image. Based on the structure of the underlying model, we have developed an algorithm that applies finite differences to accurately reflect the behavior of the geometric heat equation. Input for the method consists of a file containing 4-tuples, (x,y,z,G) , where (x,y,z) represents the 3 dimensional co-ordinates of object points and G symbolizes the gray scale level of the corresponding voxel. Using Mathematica, we have developed a tool that when provided with data from N two dimensional sections (planes) cut from a three dimensional surface, detects the edges of the object in all N 2D sections and constructs the 3D object.

YMC2004 TALK ABSTRACT

Damir Dzhilil Dzhafarov

Purdue University, West Lafayette IN

Title: On Binary Relations in Definitions of Finiteness

Description:

Definitions of finiteness are set-theoretical properties that, if the Axiom of Choice (AC) is assumed, are equivalent to the usual definition of finite set. We introduce a framework for generating definitions of finiteness in a systematical way: basic definitions are obtained from properties of certain classes of binary relations, and further definitions are obtained from the basic ones by closing them under subsets or under quotients.

We work in set theory without AC to establish relations of implication and independence between these definitions, as well as between them and other notions of finiteness previously studied in the literature. It turns out that several well known definitions of finiteness (including Dedekind finiteness) fit into our framework by being equivalent to one of our definitions; however, a few of our definitions are actually new. One of those new definitions provides a partial answer to a question regarding closure under unions, but other similar questions remain open.

YMC2004 TALK ABSTRACT

Shawn Elledge

Arizona State University

Title: Applications of Graph Pebbling to Group Theory

Description:

In 1989 Chung used graph pebbling to prove a number theoretic result of Kleitman and Lemke, first conjectured by Erdős and Lemke. Kleitman and Lemke then conjectured group theoretic generalizations of this result. Our research aims to use graph pebbling to prove these conjectures for solvable groups.

YMC2004 TALK ABSTRACT

Jim Freitag, Jennifer Mo

University of Illinois, Northwestern University

Title: Generalizations Schelling Neighborhood Segregation Model (Evolutionary Game Theory)

Description:

We have been working on a research project this summer at the University of Illinois summer NSF REU on Mathematical Modeling of Social and Political Issues. In 1971, Thomas Schelling created a simple model that showed how segregation might emerge in cities despite the absence of overt racism. His model involved two player types, black and white who played on a checker board with dimes and pennies. Each turn one player was randomly selected and could choose to move or stay put (there were empty houses into which he could decide to move). The preferences of the players were such that a player needed a fraction of his neighbors to be of his same type to be content. Below this threshold a player would be discontent. So, the individual actors in this game do not overtly want segregation, but they do want to have a certain number of neighbors of their same type. Schelling showed that even preferences of this kind could lead to segregated cities.

For our research project, we generalize Schelling's original model by moving from two player types to N player types and by playing the game with different utility functions and different concepts of who the neighbors are of a given player. We model the game using evolutionary game theory and a regularly perturbed markov process (we assume sometimes people make mistakes). We have proved some theorems about the stochastically stable states in the game in its different forms and have made computer simulations of the game as well.

Basically, we would like to talk about the nature of the stochastically stable states of the game and show some theorems on these and other aspects of the game, and we would like to show the computer simulations of the game being played in a few of the different configurations of the game. We also discuss the social aspects of our results.

For our project we drew heavily on the results of Schelling and Peyton Young on evolutionary game theory and regularly perturbed markov processes.

YMC2004 TALK ABSTRACT

Micah J. Fuerst

Wright State University

Title: Existence Status of Circulant Weighing Matrices

Description:

A weighing matrix is a $n \times n$ matrix with $\{0, -1, 1\}$ entries, that when multiplied by its transpose yields the identity matrix times some constant k , where k is the weight. In the case of my work, I am dealing with matrices that are said to be circulant, meaning that each row is a cyclic shift of the previous row. My work consists of proving the existence and nonexistence of certain circulant weighted matrices, denoted $CW(n, k)$, with given parameters n and k . This work was done almost exclusively with algebraic techniques.

Originally, these matrices were used by statisticians as mathematical instruments that could be used to precisely weigh and measure objects. In the 1960's they were used to measure anything from voltages to precise weights of objects. Today, however, they have been found much more beneficial to engineers working with satellite and digital communications. They have been found to have many similarities with perfect ternary arrays, and these arrays have been implemented in our digital communications. My work consists of merely finding these matrices, however, and handing over the given information so that it can be used by the engineers.

Our primary technique in finding these matrices through algebra. We look at these elements as group ring elements and label each element in our group of order n by $0, 1$, or -1 , and because our matrices are cyclic shifts, we only look at cyclic groups. We can then apply characters, which are homomorphisms from the group ring element into the set of all the n roots of unity. Through these techniques, we can see that all the weights must be perfect squares. We can more importantly see that when the coefficients are squared, they sum to this perfect square, but the coefficients alone sum to the square root of this perfect square. This provides for us a very effective way of eliminating possible matrices.

Yet another algebraic technique is developed through the idea of integer circulant weighing matrices. In this case we can extend our possible entries to include all entries between an a given integer, and its negative. Once again we look at these as if they were group rings. We reach them by collapsing with the use of a homomorphism from the group of order n , into a divisor of n so that we have a kernel that is an integer value, which is the same integer value that the entries can range from. Next, we use a multiplier, which can be found through various theorems, and group the elements into distinct orbits. These orbits force all the elements within it to have the same coefficients, and one can see how this would create limiting circumstances that could show nonexistence. The orbits also show us where entries will go in the circulant weighing matrix. These are but a few techniques that can be used.

The goal of this research that is being primarily done by K. T. Arasu is to eliminate as many of the unknown cases as possible. My research thus far, conducted with Dr. Arasu and graduate student Grady Burkett, has consisted of eliminating the cases from a table presented in the Ph. D. Thesis of Yoseph Strassler of Bar-Ilan University. The hope is that we can eventually notice patterns and eliminate entire families of cases, as has recently been done with weights 9, and 16. Once this has been accomplished, the next goal is to construct as many of these matrices as possible, so they can be used and implemented by engineers.

YMC2004 POSTER ABSTRACT

Juan Gallegos

University of Houston - Downtown

Title: On Estimating Survival Function of Stochastic Order

Description:

Let F , G , and H be survival functions satisfying $F \leq H \leq G$. Lee, Yan, and Shi (1999) has developed an algorithm to estimate the survival function H when F and G are unknown. However, lacking a closed form of the estimator makes the investigations of the properties of the estimator difficult. In this paper, we propose alternative estimators for H in the case where both F and G are unknown. In this paper, the estimators are proven to be strongly uniformly consistent for the estimator for both cases; the formulas for the bias and mean squares error (MSE) are also derived. In the simulations (that we have done so far) the MSE of our estimators, when F and G are known, are very close to (a little bigger than) that of Lee, Yan, and Shi, the differences are in the thousandth place. Moreover, the MSE of our estimators are uniformly better than that of the empirical survival function.

YMC2004 TALK ABSTRACT

Jeff Ginn, Jenell Nyberg

Central Michigan University

Title: New Results on Difference Sets

Description:

The study of difference sets lies at the intersection of algebra and combinatorics. Our work focuses on determining the existence of difference sets for various parameters (v, k, λ) . We use techniques from representation theory and algebraic number theory to aid the search. In particular, we examine groups of order 63, 121, and 4095.

This research was supported by an NSF grant and done under the supervision of Ken Smith at Central Michigan University.

YMC2004 TALK ABSTRACT

Chris Hammond

The Ohio State University

Title: Density of States of Random Symmetric Toeplitz Matrices

Description:

We show that if the entries of sym $N \times N$ symmetric Toeplitz matrices are chosen iid from a distribution with mean 0, variance 1, and finite higher moments, then as N tends to infinity, the distribution of eigenvalues, appropriately normalized, is independent of the distribution we started with. We use the method of moments, calculate the first few moments explicitly, and show that the moments are bounded by those of the Gaussian.

YMC2004 TALK ABSTRACT

Gwyneth Harrison-Shermoen

Wesleyan University

Title: A Search for f -Invariant ϵ -Scrambled Sets

Description:

It has been shown that ϵ -scrambled sets exist for all and only chaotic and non-uniformly non-chaotic maps. The goal of this project is to find necessary and sufficient conditions for the existence of f -invariant ϵ -scrambled sets. Du showed in his January 2004 paper that all turbulent maps have f -invariant ϵ -scrambled sets. We have shown that if a map has such a set, then it must be chaotic. We continue to look for further restrictions on the type of map that can have such a set. For example, must it have a periodic point of odd period? Periodic points for all periods?

YMC2004 TALK ABSTRACT

Benjamin Harris

Brown University

Title: The Minimum Positive Semidefinite Rank of a Graph

Description:

If $A=(a_{ij})$ is an n by n , Hermitian matrix, we may associate a graph, G , to A where $V(G)=\{v_1, \dots, v_n\}$ and $(v_i, v_j) \in E(G)$ iff a_{ij} is nonzero. Given a graph, G , the problem is to determine the minimum rank of all positive semidefinite (PSD) matrices whose graph is G . Let $\text{msr}(G)$ denote the minimum PSD rank of G . New results to be presented will pertain to characterizations of graphs with msr equal to 2, 3, and $V(G) - 2$, the msr of the join of graphs, and the relationship between $\text{msr}(G)$ and $\text{msr}(G \setminus v)$ for a given v in $V(G)$.

YMC2004 TALK ABSTRACT

John Hegeman

Stanford University

Title: Polygonal Designs: Existence and Construction Results

Description:

Polygonal designs are introduced as a generalization of combinatorial 2-designs and as a specialization of partially balanced incomplete block designs. We develop enough theory to reveal the structure, and thus, to resolve the existence problem for several families of polygonal designs, and derive necessary conditions for general cases. We will discuss several construction methods including a generalization of the recursive construction for combinatorial 2-designs.

YMC2004 TALK ABSTRACT

Aaron Thomas Hill

Brigham Young University

Title: Construction of Semi-symmetric Graphs of Girth 6 and Degree 4

Description:

While at an REU at Northern Arizona University I discovered (with the help of my advisor Dr. Steve Wilson) a new way to generate worthy semi-symmetric graphs of girth 6 and degree 4 (from smaller symmetric graphs of girth 3). This yeilds an infinite family of semi-symmetric graphs. The smallest previously unknown example this yeilds is one with 120 vertices and 240 edges that resulted from the construction with the base graph a dodecahedron.

YMC2004 POSTER ABSTRACT

Daisy (Yan) Huang

UC Berkeley

Title: Estimating Survival Curves with Boundary Constrains

Description:

Stochastic ordering of survival functions is an important concept and has been studied by predecessors, such as Rojo, Dykstra, Lee, Yan and Shi. In "Nonparametric Estimation of Bounded Survival Functions with Censored Observations," Lee, Yan and Shi proposed a nonparametric maximum likelihood estimator (NPML) for the survival function with the boundary condition, such that, the estimated survival function is bounded by two other known survival functions. However, this NPML has no closed form expression and therefore makes further investigations, such as the consistency and the mean square error, of this estimator difficult. Inspired by the studies described in Rojo's papers, we proposed an estimator of a closed form expression for this problem, and will prove that our estimator is a better one. We will also look into the case in which the two survival functions for the boundaries are unknown and are needed to be estimated by drawing random samples from these two distributions, and will propose an estimator for this case.

YMC2004 TALK ABSTRACT

Colleen Hughes

Denison University

Title: Straight Line Embeddings of K_6

Description:

Any embedding of the complete graph with six vertices, K_6 , will have at least one pair of linked triangles, not necessarily constructed of straight lines. In this talk we explore the possibility of constructing straight-line embeddings of K_6 with 1, 3, 5, and 7 pairs of triangles respectively.

YMC2004 TALK ABSTRACT

Johnson Xin Jia

University of California, Los Angeles

Title: Hypberbolic Isometries on R-trees

Description:

I have been studying free actions by a group on R-trees. The main motivation is a new result by Ilya Kapovich, et al. showing that translation equivalent words in a free group have the same Whitehead graphs under arbitrary group automorphisms. The main focus of my research is on finding words that are translation equivalent. I also attempted at investigating the geometry on R-trees given a group of hyperbolic isometries.

YMC2004 TALK ABSTRACT

Benjamin Johnson

Kenyon College

Title: Searching for New Error Correcting Codes

Description:

The theory of error-correcting codes is used to improve the reliability of digital communications. One of the main problems in this area is the construction of efficient codes. Algebraic coding theory uses algebraic tools to achieve this goal.

In this work, we focus on a class of codes called Quasi-twisted (QT) codes. Taking advantage of the rich algebraic structure of these codes, we have devised improvements to existing computer algorithms which search for new codes. The improvements are based on the Chinese Remainder Theorem. We are currently implementing these algorithms, and we hope to find some new record-breaking codes. We will be running our search

code on the computers at OSC, the Ohio Supercomputer Center at Ohio State University. Undergraduate abstract algebra is the only prerequisite to understand this talk.

YMC2004 TALK ABSTRACT

Alicia Jumper

Oakwood College (The Ohio State University)

Title: Mathematical Models of the Human Sleep/Wake Cycle

Description:

In this research, we study models for the human sleep-wake cycle. In particular, we consider the Kronauer X-Y model for the circadian rhythm and the Ultradian Two Process model for REM and non-REM sleep. These models take the form of systems of ordinary differential equations and both dynamical systems and computational methods are used to analyze them. These models are used to study how different patterns of light may reset the sleep-wake cycle and how desynchronized rhythms arises during free-running conditions.

YMC2004 TALK ABSTRACT

Nathan Kaplan

Princeton University

Title: Non-Unique Factorizations in Numerical Monoids Generated by Intervals

Description:

Given a numerical monoid $S = \langle n_1, n_2, \dots, n_t \rangle$ we have investigated $\delta(S)$, the set of all possible consecutive differences between lengths of factorizations of consecutive elements, and for an element m of S we look at $\rho(m)$, the longest possible factorization length of m divided by the shortest factorization length.

My research has focused in part on monoids generated by intervals, $S = \langle n, n+1, \dots, n+x \rangle$, and has shown among other things that in this case $\delta(S) = 1$, and produced an expression for the smallest elasticity greater than 1 that appears as the elasticity of some element in S . When $x = n-1$, so that $S = \langle n, n+1, \dots, 2n-1, 2n, \dots \rangle$ S contains all integers greater than $n-1$, we can say a ton about elasticities. It is easy to show that the maximum factorization length of an element in this case is $(2n-1)/n$, so given a fraction $1 < q < (2n-1)/n$, we now have an expression the number of elements m in S that give $\rho(m) = q$.

More recently I have derived an expression that solves the membership problem for the monoid $S = \langle n, n+x \rangle$ and hope to generalize it further in the coming weeks. There is a wide range of things that I could talk about related to my research but I think I would like to focus in on monoids generated by intervals. I would talk about the proof that their delta sets are always $\{1\}$, which is really cool, and interesting properties of the expression we came up with for the elasticity of an element in such a monoid.

YMC2004 TALK ABSTRACT

Matthew King

Title: Two size PBD\'s with a Maximum Number of Triples

Description:

The covering problem gives a solution of construction a PBD(v , $\{3, 2\}$, 1) with a maximum number of triples. In this note, we will find the necessary and sufficient conditions to construct a PBD(v , $\{3, k\}$, 1) with $k = 4$ or 5 having a maximum number of triples. Note that the case for $v \equiv 5 \pmod{6}$ and $k = 5$ is already known. It is known that the minimum number of blocks of size k where $v \equiv 1, 3 \pmod{6}$ is 0 , but we will also investigate the possibility of designs of this nature with the nonzero but minimum number of blocks of size k .

YMC2004 TALK ABSTRACT

Steven Klee, Leah Yates

Valparaiso University, East Carolina University

Title: Tight Subdesigns of the Higman-Sims Design

Description:

Abstract: The Higman-Sims design is an incidence structure of 176 points and 176 blocks of cardinality 50 with every two blocks meeting in 14 points. The automorphism group of this design is the Higman-Sims simple group. We demonstrate that the point set and the block set of the Higman-Sims design can be partitioned into subsets

X_1, X_2, \dots, X_{11} and B_1, B_2, \dots, B_{11} , respectively, so that the substructures (X_i, B_i) , $i = 1, 2, \dots, 11$, are isomorphic symmetric $(16, 6, 2)$ -designs.

YMC2004 POSTER ABSTRACT

Mary E. Kloc

Kenyon College

Title: Computational Modeling of Ca^{2+} Oscillations in Astrocytes

Description:

Calcium oscillations have numerous functions in a wide variety of cells and are integral to the regulation of many biological processes. Their known functions range from the regulation of heart beats, circadian rhythms, and hormone secretion to roles in the fertilization and wound healing processes. They are also present in other cells, such as astrocytes, in which their function remains unknown.

Astrocytes are the most abundant cell type in the central nervous system. They are situated next to neurons and make up the blood brain barrier. Considering the importance of Ca^{2+} oscillations in such a diverse array of cells throughout the body, their presence alone in astrocytes suggests that they must have a function there as well.

This study utilizes the astrocytic calcium system that we have previously explored. The model consists of a system of three differential equations, which describe $[\text{Ca}^{2+}]$ oscillations in the cytosol, mitochondria, and endoplasmic reticulum of astrocytes, which are glial cells in the central nervous system. Numerical integration, using the Gear algorithm, is used to produce concentration vs. time data that can be compared to experimental data. $[\text{Ca}^{2+}]_{\text{cyt}}$ vs. $[\text{Ca}^{2+}]_{\text{ER}}$ vs. $[\text{Ca}^{2+}]_{\text{mit}}$ can be plotted to produce three dimensional phase portraits. Comparison of these plots reveals the large change from chaotic to simple periodic behavior when only a single parameter (most notably, the fractional external stimulation) is changed; this is consistent with experiment. It is known that dynamical systems in a chaotic regime, such as $[\text{Ca}^{2+}]$ oscillations in astrocytes, can tie themselves in knots in 3-D space. We are probing for knots in chaotic phase planes and using the principles of knot theory to tie the governing mathematics to the biological behavior.

The most important concept from the field of knot theory to be applied in this project is that the fundamental topological properties of a knot must be invariant over continuous transformations. Most relevantly to this case, the unknot cannot be continuously transformed into any other kind of knot.

The mathematics of knot theory may prove to be very important when applied to dynamic biological systems. From a biochemical point of view, the statement above implies that a periodic (unknotted) state cannot move to a chaotic (knotted) state through a continuous transformation; we want to show that certain types of behavior are invariant under this set of topological principles. We will demonstrate that periodic behavior of any period is topologically equivalent to any other periodic behavior, but is not topologically equivalent to chaotic behavior. We plan to characterize aspects of this biological system according to the mathematical operations and transformations that are permitted by the rules of knot theory; find biological parameters for which the system is invariant to a change in value; and gain further insight into how system moves from a chaotic to a periodic regime when a continuous transformation of this type is not possible.

YMC2004 POSTER ABSTRACT

Lorraine Kraus

The College of New Jersey

Title: Multiple Attacks on Methods for Hiding Messages

Description:

The research is aimed at showing the vulnerability in the system developed by J.R. Miotke and L. Rebollo - Neira to hide messages in their paper, Oversampling of Fourier Coefficients for Hiding Messages, from Applied and Computational Harmonic Analysis, 2004. We came up with two ways of breaking the encryption scheme which can be used for chosen plaintext, known plaintext, or chosen ciphertext attacks, and which depend on whether the signal, or garbage function, changes with each encoded message. We then carried out the attacks electronically and analyzed the errors between the original and reconstructed messages resulting from different garbage functions and varying numbers of Fourier coefficients.

YMC2004 POSTER ABSTRACT

Mollie Lisle

Pacific Lutheran University

Title: A Quantitative Study of Student Retention by both Statistical and Probabilistic Approaches

Description:

Many institutional researchers have conducted various qualitative and quantitative studies on the characteristics of students in order to develop effective student retention programs. Under Dr. Dane Wu's supervision, last summer Kerri Fletcher and Lindsey Olsen conducted a quantitative study of student retention using eight years of data at Pacific Lutheran University. Through the use of attributable risk, four top risk factors for retention were identified and ranked. Then an issue of confounding risk between two risk factors was addressed by the means of 'relative attributable risk'. This study led to the following further explorations that a team of three, Dr. Wu, Mark Westland, and myself, Mollie Lisle, are tackling this summer. (1) Use more sophisticated stratification for single risk factor. (2) Study stratification of two risk factors. (3) Apply regression technique to identify risk factors for comparison to the probabilistic approach. (4) Apply our method into more general data other than PLU's.

YMC2004 TALK ABSTRACT

Micah Miller

Bowdoin University

Title: Constructing Minimal Length Representatives In Thompson\'s Group F

Description:

Using the standard finite presentation for Thompson\'s group F, we describe a method for producing minimal length representatives for a large class of elements of Thompson\'s group, generalizing the method exhibited by Cleary and Taback. We discuss possible methods for constructing minimal length representatives for every element in Thompson\'s group.

YMC2004 TALK ABSTRACT

Thomas Callaghan

Georgia Institute of Technology

Title: Random Walker (Monkey) Rankings for College Football

Description:

The Bowl Championship Series (BCS) National Championship for NCAA Division I-A football is currently determined by the outcome of a single bowl game played between the #1 and #2 teams in the BCS Standings, which includes both polls and ranking algorithms. The difficulty in selecting these two teams lies in the fact that each of the 117 Division I-A teams plays only approximately 11 regular season games. Using only the win-loss information of those games, we develop a family of rankings defined in terms of independent random walks on the graph of teams (vertices) and games played (edges), in one parameter (the bias of the walk). The virtue of this ranking system lies in the simplicity of its explanation. We compare results for recent seasons against BCS Standings and its components, demonstrating that the results from this simple algorithm is competitive with more complicated methods. We investigate the statistical properties of the randomly walking voters and examine the asymptotic properties of the rankings at extreme values of the single parameter. We also investigate the connection between the rankings and the underlying community structure of the network of games played.

YMC2004 TALK ABSTRACT

Jessica J. Poole, Misha Teplitskiy

Texas Southern University, Rice University

Title: Modular Pascal Triangle

Description:

This project examines the congruence classes of binomial coefficients to a prime square modulus as given by a fractal generation process for lattice path counts. The process depends on the isomorphism of partial semigroup structures associated with each iteration. We also consider integrality properties of certain critical coefficients to arbitrary arguments, instead of just to the prime arguments appearing in their original function, it transpires that integrality of the coefficients is indicative of the primality of the argument.

YMC2004 TALK ABSTRACT

Jesse Raab

College of Charleston

Title: Slightly Irregular Non-Planar Graphs

Description:

We have continued the work of Limaye, Sarvate, Young (2003) and Limaye, Sarvate, Stanica (2004) by studying graphs whose degree sequence fails to be regular by only a few entries. As we complete our investigation our theorems will be used in making a program which, when given an arbitrary degree sequence, will draw a planar representation of a graph with that degree sequence if possible, and otherwise will draw a non-planar representation.

YMC2004 TALK ABSTRACT

Daniel Reiser

New Mexico State University

Title: Factorization Lengths in Numerical Monoids

Description:

A (primitive) numerical monoid is a subset S of the nonnegative integers that includes 0, is closed under addition, and satisfies $\gcd(S) = 1$. If n_1, \dots, n_t are positive integers satisfying $\gcd(n_1, \dots, n_t) = 1$, then the numerical monoid generated by n_1, \dots, n_t is the set of all linear combinations of n_1, \dots, n_t with nonnegative integer coefficients. It is a fact that every numerical monoid has a unique finite minimal system of generators $\{n_1, \dots, n_t\}$.

An obvious fact about these monoids is that if $t > 1$, then factorizations into irreducibles are non-unique. Thus, we can study the set of all lengths of factorizations of a given element. If m is in the numerical monoid S and $\{x_1, \dots, x_n\}$ is the set of lengths of factorizations of m in S , where $x_1 < x_2 < \dots < x_n$, then the delta-set of m is $\Delta(m) = \{x_i - x_{i-1} : 2 \leq i \leq n\}$. The delta-set of S , $\Delta(S)$, is the union of the sets $\Delta(m)$, where m ranges over all elements in S .

We have several interesting results about $\Delta(S)$. First, if S is generated by $\{n, n+k, \dots, n+tk\}$ for some positive integers n, k , and t , then $\Delta(S) = \{k\}$. If S is generated by $\{n_1, \dots, n_t\}$, then $\min \Delta(S) = \gcd(\{n_i - n_{i-1} : 2 \leq i \leq t\})$. We have a way to compute the maximum element of $\Delta(S)$ in finite time. For any positive integers t and k , we can construct a numerical monoid S such that $\Delta(S) = \{k, 2k, \dots, tk\}$. However, there exist delta-sets with "jumps" such as $\Delta(S) = \{1, 2, 3, 5\}$ and $\Delta(S) = \{2, 4, 6, 8, 14\}$.

YMC2004 TALK ABSTRACT

Nancy Rodriguez

University of San Diego

Title: Displacement Analysis on a Three-Layered Elastic Strip Subjected to Heaviside Loading

Description:

Our society strongly depends on bridges, buildings, electrical systems, and airplanes. Therefore, an essential thing to consider when creating a design is the risk of structural failure. As a relevant factor, we study in this work the displacement caused by the stress wave propagation in a three-layered elastic strip subjected to Heaviside loading.

Based on Hook's law, we use two different approaches to determine the value of the displacement at any given time and position. The first approach involves stress formulas obtained by solving a system of difference equations. The second approach is an algebraic approach that involves Laplace transformations. We further discuss the advantages and disadvantages of both methods presented, and support our analytical results with graphical output.

YMC2004 POSTER ABSTRACT

Sidharth Rupani

Worcester Polytechnic Institute

Title: Modeling iBOT Belt Dynamics

Description:

Note: I would be comfortable with any one, either giving a talk or doing a poster presentation.

Abstract: The iBOT, a revolutionary personal mobility device developed by DEKA Research and Development Corporation, contains several belts that transfer mechanical power through the system. The dynamic characteristics of the belts are obviously important to the operation of the iBOT. Our task is to create a mathematical model of some aspects of the belt dynamics. Partial differential equations and Lagrangian dynamics are employed to understand the elements of this electro-mechanical system. Ultimately, finite difference numerical methods are applied to solve the rather complicated system of partial differential equations developed.

YMC2004 TALK ABSTRACT

Peter Schallot

Slippery Rock University

Title: Trajectory estimation of small diameter rockets

Description:

The purpose of this research is to develop an accurate and efficient method for estimating the trajectory of small diameter rockets based on accelerometers and gyroscopes mounted to the flight vehicle. Specifically, we will derive the governing equations for the trajectory of a 2.75-inch rocket typically launched from a helicopter platform. Eulerian and quaternion formulations will be presented along with consideration for wind effects, sensor errors, and mass variations as time permits. Simulations based on sensor data obtained from the U.S. Army will also be provided.

YMC2004 TALK ABSTRACT

Gagan Sekhon

California State University, Hayward

Title: Time Release Cryptography

Description:

Time Release Cryptography using a Cryptographic Beacon Time release cryptography is the notion of sending messages into the future: some information needs to be made available to a set of people simultaneously. For example, insider trading laws disallow critical information that may affect the rise or fall of a certain stock being released to certain parties ahead of other. If this information is sent at the time of release delays can be caused due to peak network traffic. However, another solution is using time-release cryptography by sending an encrypted version of the information ahead of time and release the decryption key when the information needs to be made available. Since the key is simply a value and can be mirrored on multiple websites across the internet there will be no delays in accessing it. This allows everyone access to the information simultaneously and no one is able to access the information before it is explicitly made available. The approach that we used here is of a sequence of numbers generated by a one-way function. This makes the calculation of values released in the past easy; however it is extremely difficult to calculate values to be released in the future. Individuals who wish to encrypt information for future simultaneous disclosure are given an encrypted future value to be used as a key. This ensures that the future terms in the sequence are not known before their release; however they can still be used as part of the encryption process.

YMC2004 TALK ABSTRACT

James Owen Sizemore

UC Berkeley

Title: Relations Between Crosscap Number and Genus of Torus Knots

Description:

In 1978 B.E. Clark defined crosscap number, $c(K)$, of a knot to be the minimal genus of all non-orientable surfaces which span the knot and gave an upper bound for the number in terms of the genus: $c(K) \leq g(K) + 1$, where $g(K)$ is the genus of the knot. The obvious next question is whether an upper bound for the genus exists in terms of the crosscap number. In this talk I will exhibit a family of torus knots that shows that this cannot be the case and in so doing provide an answer to an open question posed by Colin Adams. Also I conjecture $c(K) \leq \lfloor (cr(K) + 4)/12 \rfloor + 1$ for a torus knot where $cr(K)$ is the crossing number and $\lfloor \cdot \rfloor$ denotes the greatest integer function. I prove the conjecture for (p,q) torus links such that either p or q is even.

YMC2004 TALK ABSTRACT

Tyler Smith

University of Illinois at Urbana-Champaign

Title: l -Groups Acting on $\text{CAT}(0)$ Cube Complexes

Description:

An l -group is a group which admits a lattice ordering where multiplication on the left and right act as lattice automorphisms. We exhibit that for a certain subclass of l -groups there are $\text{CAT}(0)$ cube complexes on which the groups act properly discontinuously. The complex arises from Sageev's halfspace construction \cite{Sag95} when considering halfspaces to be prime ideals and prime filters of the l -group. We examine the structure of the space in relation to the lattice and also when the action is cocompact, leading to a biautomatic structure on the l -group \cite{NR98}.

\bibitem{NR98} G.~A.~Niblo and L.~D.~Reeves. The geometry of cube complexes and the complexity of their fundamental groups. {\em Topology}, 37(3):621-633, 1998.

\bibitem{Sag95} Michah Sageev. Ends of group pairs and non-positively curved cube complexes. {\em Proc. London Math. Soc. (3)}, 71(3):585-617, 1995.

YMC2004 TALK ABSTRACT

Sucheta Soundarajan

The Ohio State University

Title: m-paths of the $3x+1$ problem

Description:

The $3x+1$ conjecture is as follows: Consider the function $f(x)$ defined on the set of positive integers not divisible by 2 or 3, with $f(x)=3x+1/(2^k)$, where k is the largest integer such that $f(x)$ is still an integer. It is believed that for every positive number n , there is some number j such that j applications of f to n produces the value 1. An m -path of a number n is the sequence of m numbers k_1, \dots, k_m such that k_i is the value of k in the function definition in the i -th iteration of the function originally applied to n . The research that my partner and I have been working on is looking at 'bad' m -paths: m -paths which could potentially correspond to a number n which might never reach 1.

YMC2004 POSTER ABSTRACT

Leya Tesmenitsky

Hobart and William Smith Colleges

Title: A Game Based on Vertex-Magic Edge Labeling

Description:

Consider a game based on labeling vertices and edges of graphs. For a given graph G , let V be the number of vertices and E be the number of edges. Let the labels available be the numbers in the set $\{1, 2, \dots, V+E\}$. Two players take turns labeling vertices and edges in the graph. Labels on an edge or vertex cannot be repeated nor can an edge or a vertex be labeled more than once. The first time a vertex and all the edges incident to it are given labels, the sum of those labels becomes the magic constant k . From that moment on, any time a vertex and the edges incident to it are all labeled, the sum of the labels must be k . It is possible that some of the edges or vertices may be forced to be left unlabeled. The goal of the game is to be the last person to make a move on the graph. While the game can be played on any graph, if a graph has certain properties, a particular player can have a winning strategy. I have been investigating such games and will present new strategies I have developed.

YMC2004 TALK ABSTRACT

Benjamin Vugteveen

Grand Valley State University, MI

Title: You, Me, and Semi-Symmetry: Disjunction Graphs and Their Properties

Description:

The topic of semi-symmetric graphs is an area of current research in algebraic graph theory. In this talk, we will discuss a general construction of semi-symmetric graphs, using the bi-transitive disjunction graphs $D_N(a,b)$, and then show what very minimal conditions imposed on pairs of these graphs will give us semi-symmetric products.

YMC2004 POSTER ABSTRACT

Mark Westland

Email: westlama@plu.edu

Pacific Lutheran University

Title: A Quantitative Study of Student Retention by both Statistical and Probabilistic Approaches

Description:

Many institutional researchers have conducted various qualitative and quantitative studies on the characteristics of students in order to develop effective student retention programs. Under Dr. Wu's supervision, last summer Kerri Fletcher and Lindsey Olsen conducted a quantitative study of student retention using eight years of data at Pacific Lutheran University. Through the use of attributable risk, four top risk factors for retention were identified and ranked. Then an issue of confounding risk between two risk factors was addressed by the means of 'relative attributable risk'. This study led to the following further explorations the Dr. Wu, Mollie, and I are taking on this summer. (1) Use more sophisticated stratification for single risk factor. (2) Study stratification of two risk factors. (3) Apply regression technique to identify risk factors for comparison to the probabilistic approach. (4) Apply our method into more general data other than PLU's.

YMC2004 TALK ABSTRACT

Phillip Whitman

University of Texas at Austin

Title: Analyzing behavior of solutions of a nonlinear elliptic PDE

Description:

I will present some results on solutions of $\text{Laplacian}(u) = u^{-a}$, where $0 < a < 1$. I will define a "tornado sequence" of solutions and outline how to use fixed point theorems and the maximum principle to rule them out. I will generalize these results for the equation $\text{Laplacian}(u) = g(u)u^{-a}$, where g satisfies some weak conditions. This result implies continuity of limits of positive solutions for dimension 2.

YMC2004 TALK ABSTRACT

Christopher D. Wreh, II

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Texas Tech University

Title: A Simple Model for Contaminant Transport

Description:

Modeling a chemical or biological attack on a building requires us to understand how contaminants are transported throughout the structure. We must understand and model the airflow in the building. We will assume that the attack does not affect this flow, but the flow transports a contaminant to other parts of the building. In this paper, we discuss a simple Constantly Stirred Tank Reactor (CSTR) model for the transport of a contaminant. We will make simplifying assumptions concerning the details of the airflow and the mixing of the contaminant. For instance, we will assume that the contaminant is equally dispersed throughout a given room by volume and that the flow into a room is equal to the flow out of the room (no pressure change). These assumptions will lead us to a simple ordinary differential equation (ODE) system describing the change of contaminant (by weight) in each room. Following are more details of what we are interested in including the main ODE.

YMC2004 TALK ABSTRACT

Inna Zakharevich

Harvard University

Title: A Generalization of Wigner's Semicircle Law: Distribution of States of Random Symmetric Matrices

Description:

We present a generalization of Wigner's semicircle law. We consider a sequence of probability distributions, (p_1, p_2, \dots) each of which has mean zero and finite higher moments. If we take an $N \times N$ real symmetric matrix with entries independently chosen from p_N , we can analyze the distribution obtained by putting a delta-mass at each eigenvalue. If we normalize this distribution by $\sqrt{N \cdot \text{second moment of } N\text{th distribution}}$ we will show that as N approaches infinity for certain sequences $\{p_N\}$ the distribution will weakly converge to a universal distribution. We will give a formula for the moments of the universal distribution in terms of the rate of growth of the k -th moment of p_N (as a function of N), as well as some implications of the formula. As a corollary we obtain Wigner's semicircle law: if all moments of a distribution are finite, the distribution of eigenvalues is a semicircle.

YMC2004 TALK ABSTRACT

Liya Zhu

UC Berkeley

Title: Extraction Degree of Zero Sequences

Description:

Kim Cevello (New York State University at Geneseo), Denise Terry (University of Redlands), and I did our research on Zero Sequence of Finite Abelian Groups at Trinity University, in San Antonio, Texas. We investigated properties of the extraction degree, which is defined to be the supremum of β over α such that β copies of X is a subsequence of α copies of Y , X and Y both zero sequences. Kim Cevello will have her introductory talk on the general definitions and preliminary results, and I will cover more topics on other theorems.