Spectral Reconstruction and Solution of PDEs Using Freud Orthogonal Polynomials

William S Rosenthal
Arizona State University [Mentor: Anne Gelb]

Abstract of Report Talk: Orthogonal polynomials defined on [-1,1] are often used in the spectral reconstruction of sufficiently well-behaved and analytic functions. They are commonly employed in numerical simulations of partial differential equations and image reconstruction. If imaging data is equally spaced and periodic, Fourier methods are typically used. However, these do not work for non-periodic and piecewise-smooth functions as a result of the Gibbs phenomenon. Filtering is commonly applied but causes blurring at the internal boundaries of the image. We seek a high-order approximation method that is not affected by the Runge effects that result from interpolating on equally-spaced points. A suitable solution is to use a least-squares method that does not require a lot of points. Our present study uses polynomials defined by the Freud weight function to calculate a spectrally-accurate approximation to a piecewise smooth function. We investigate the accuracy of this approximation and compare it to splines. We also investigate how Freud polynomials may be used in solving non-periodic partial differential equations on equally spaced grid points. In particular, we show how to improve the CFL number. However, the Freud approximation is inaccurate near the boundaries. By examining the weight function and corresponding Lebegue constant for our method, we calculate the region where the Freud approximation is sufficient, and where another method should be used to restore spectral convergence over the entire domain. [RW14194603]

Contact: `william.rosenthal@asu.edu`
Received: July 22, 2006