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A Computational Study of the Quantization of Billiards with Mixed Dynamics

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Abstract of Report Talk: A Computational Study of the Quantization of Billiards with Mixed Dynamics

This project is a numerical study of the quantization of billiards with mixed dynamics, that is, particle in a box models whose box corresponds to a classical Hamiltonian system that may display either chaotic or integrable behavior, depending upon initial conditions. Our problem is closely related to the classic question in spectral theory popularly known as 'Can you hear the shape of a drum?' That is, can one exactly describe the boundary of a plane region from a list of the eigenvalues of functions on that region satisfying Dirichlet boundary conditions (that is, the functions vanish on the boundary). More specifically, we are interested in the eigenvalues corresponding to families of solutions to the Helmholtz equation on a mushroom-like planar geometry with Dirichlet boundary conditions for very high wavenumbers. A consecutive list of eigenvalues yields a cumulative level spacing distribution that describes the dynamics of the system. A perfect Poisson distribution corresponds to a perfectly regular dynamics, a Wigner distribution to completely chaotic dynamics, and an intermediate distribution to mixed dynamics. This investigation is currently in progress and employs various numerical techniques. The end goal of this research is to better understand how the geometry of a quantum billiard influences its dynamical behavior and thus gain deeper insight into the quantization of Hamiltonian systems with mixed chaotic/integrable dynamics. [WW04174331]

[Joint work with Dr. Alexander Barnett (Dartmouth)]

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