Ross Program for Teachers 2008: APPLICATION PROBLEMS.

Investigate one of the following two problems, and include your comments with the rest of the application.

The purpose of this section is to point out a couple of open-ended math problems we thought were fun to work on and interesting to solve. You are invited to investigate the patterns, make guesses, and look for explanations. (Complete or elegant solutions are not expected!)

NUMBER PROBLEM.

Call a number "nice" if it can be expressed as a sum of two or more consecutive positive integers. For example, 5 and 6 are nice numbers because $5 = 2+3$ and $6 = 1+2+3$.

(a) Which numbers from 1 to 50 are not nice?

What's the pattern of nice numbers for sizes beyond 50?

(b) Explain why the pattern you observed holds true generally.

(c) Find all the ways to express 1000 as a sum of consecutive positive integers.

GEOMETRY PROBLEM.

Two clocks hang on the wall, each with a second-hand rotating smoothly. Let C be the curve traced by the midpoint of the line segment connecting the tips of those second-hands. If the clocks are identical then that curve is a circle.

(a) How does curve C change if one of the clocks is larger than the other?

(b) How does it change if one of the clocks is moved to a different spot on the wall?

(c) How does it change if one of the clocks is set some time ahead of the other?