Vertical Integration in the Mathematical Sciences at The Ohio State University

1 Vision, Objectives, and Anticipated Impact

In this proposal, we describe a plan to realize at Ohio State the twin goals of the VIGRE program: (a) to broaden the preparation of undergraduates, graduate students, and postdocs in the mathematical sciences; and (b) to initiate, improve, and sustain education activities which stimulate teaching, learning, and research interactions across the boundaries of academic age, professional standing, and mathematical discipline.

We give an overview here of the parts of this plan:

Invitation to Research: In their first year, VIGRE graduate trainees will take a full year course which will consist of many short lecture series, taught by different faculty: in the fall, by senior faculty, but including junior faculty and VIGRE postdocs as the year progresses. The aim of these short courses is to provide first-year graduate students with a glimpse into the various areas of mathematics in a non-technical way and in an informal setting, and to introduce them to the basic ideas, examples, and problems that motivate research in various fields. We expect that this will be a stimulating challenge for the faculty involved. By the end of the year, the students will have seen a sizeable percentage of the Department's faculty, and will have an overview of many parts of mathematics and some feeling for the possible directions of research. One of the tasks of the faculty who contribute to this course will be to give the students references on where to go next to find out more: articles, texts, courses, and Working Groups (see below). We expect most if not all first-year graduate students to take this course, and we expect that it will attract some of our honors undergraduates as well.

Working Group Rotations: In the second year each VIGRE graduate trainee (and any other interested graduate students) will join a Working Group. These Working Groups are small groups of faculty and students engaged in a common research venture. Several faculty organize a Working Group around an interesting problem or subject, and agree to involve undergraduate and graduate students in the investigation of that topic. The format will be very flexible, and will allow for quite varied experiences: some groups may be able to involve the students in current research; others may function more like a learning seminar. At the end of the quarter, each graduate student will write up a project report and give an oral presentation. Each VIGRE graduate trainee will rotate through three different Working Groups, one per quarter, and will thus become acquainted with different areas of mathematics, different research styles, and different faculty and students as the year progresses.

Professional Development: To round out the professional development of the VIGRE trainees we will offer courses in teaching methods and skills (this is already a standard part of our TA training), computer skills, writing mathematics, delivering seminar talks, proposal writing, and putting together an application. We also will run a seminar in industrial mathematics, providing students and postdocs with experience in how and where mathematical expertise is used outside of academia; and we have identified opportunities for summer internships in industry, which graduate

students and postdocs may apply for.

Advising and Mentoring: We have in place a committee to advise and mentor our graduate students; each student is given information on the research interests of the faculty and each student's progress is reviewed yearly. We have a few suggestions for improving the current system.

Pipeline Programs: We have in place two excellent programs that contribute to the mathematical training and experience of undergraduates: the Ross Young Scholars Program (which begins with high school students, but involves students and faculty at all levels) and the Honors Program (which is part of our undergraduate mathematics major program). We discuss ways for the students in these programs to participate in research activities (among them the Working Groups described above), and we propose a conference to bring together participants in REU programs around the country. Finally, we propose a support seminar that would allow us to admit students from our applicant pool whose applications show evidence of potential but lack some of the required background for graduate work.

A Teacher Component to the Ross Program: This year, five high school teachers were included in the Ross Program, to introduce them to the "exploration method" on which the Ross Program is based. These teachers plan to try this method in their own classes, and to report back to us on their experiences. We plan to expand this new component of the Ross Program to a class of 15 participants.

The personnel to be supported or partially supported by VIGRE funds in the above outline are

- 6 VIGRE graduate trainees per year for 5 years (30 in all)
- 4 VIGRE postdocs (with three-year appointments) per year for 5 years (12 in all)
- 20 VIGRE undergraduate trainees (Undergraduate Research Experience) and 30 undergraduates (Undergraduate Research Conference) per year
- 15 high school teachers (participation in the Ross Program)

The 12 VIGRE postdoctoral positions will replace our current 9 Ross Assistant Professorships, which have a lower salary and and higher teaching load.

The aim of these initiatives is to introduce both our undergraduate students (especially honors students) and our graduate students to the research environment at a much earlier stage, and to acquaint them with a greater number of faculty. We also aim to increase the interactions between undergraduate students, graduate students, postdocs, junior faculty, and senior faculty. Finally, we want to better prepare our graduate students for academic and non-academic employment.

Among the results we expect from these initiatives is an increased effectiveness of our graduate program: we hope to see a lowering of the time required to obtain a PhD, and we also hope to see better employment placement—whether in academia or industry—for our students after graduation. Finally, we hope that our students will simply be better educated when they graduate, with a broader perspective on the extraordinary variety and range of applications of mathematics.

Some of the sections that follow describe new proposals, while others describe a feature of our Department which is already in place. To clearly distinguish between the new and the old, we mark the title of the sections where something new is being proposed with a superscript $^{\text{NEW}}$.

2 Description of the Primary Components

2.1 Overview of the Department

Ohio State is a large public land grant university with strong commitments to research, teaching, and public outreach, commitments shared by the Department of Mathematics.

While the university has traditionally been an open enrollment school, selectivity in undergraduate admissions has increased steadily and deliberately over the last ten years. Due to general education requirements for an entering class of about 6,000 freshmen, the high demand for service courses for science and engineering majors, and the needs of our own undergraduate and graduate programs, the Department of Mathematics teaches approximately 30,000 students per year. The total teaching effort amounts to roughly 6% of the University's total credit hours of instruction. To meet its teaching obligations, Ohio State must have a large mathematics department.

Currently, there are 68 full time tenured or tenure track faculty. There are 21 three-year postdoctoral positions. The graduate program has approximately 100 students, roughly 40% domestic and 60% international. There are about 230 undergraduate math majors currently enrolled. The Department employs a staff of 26 full time employees to support its research, education, and service missions.

Research. Faculty research spans most areas of modern mathematics from core or pure mathematics to interdisciplinary or applied mathematics to mathematics education. The Department is committed to building on strength in each of these three broad areas. As part of Ohio State's Selective Investment Program, the Mathematics Department was recently chosen on a competitive basis to receive significant central funding to hire senior faculty in the areas of number theory, global analysis, mathematics education, mathematics and biology, and mathematics and computation.

A significant resource for the support of faculty research is the Mathematics Research Institute (MRI). The MRI is an institute internal to the Department which supports weekly seminars, short term visitors, distinguished visitors, special emphasis quarters, and conferences.

Postdoctoral Fellows. We have 21 three-year postdoctoral positions consisting of 12 Hans Zassenhaus Assistant Professors (ZAPs) and 9 Arnold Ross Assistant Professors (RAPs). The Zassenhaus positions have been in existence for about 20 years and have salary and teaching loads competitive with named postdoctoral positions at peer research institutions. Four years ago, the Department recognized that many promising young mathematicians were unable to find suitable academic appointments after the PhD. At the same time, the Department was spending significant resources employing part time lecturers to fulfill our teaching obligations on a course by course basis. Accordingly, a second line of postdoctoral positions, the Ross Assistant Professors, was created by bundling together the funds used to hire part time lecturers and by identifying new funds for benefits to provide the stability of a three-year appointment. On the one hand, RAPs have lower salaries and higher teaching loads than ZAPs, being roughly comparable to those of senior lecturers on the Columbus campus or adjunct faculty on regional campuses of Ohio State University. On the other hand, the RAP positions fill a need by providing a means to further the teaching and research

careers of successful candidates.

Graduate Program. Like most institutions around the country, Ohio State has experienced a decline in the size of its graduate program over the last ten years from roughly 170 students to roughly 100 students. The Department's goal is to slowly increase the graduate program to 120, a good compromise between the intellectual needs of the Department, the instructional needs of the institution, and the capacity of the job market. Most students are supported as TAs, with TA stipends having risen in response to competitive pressures. Almost all students find employment within a year of graduation, roughly half in academic institutions or government agencies and the other half in private industry.

Undergraduate Program. The undergraduate major has several tracks in addition to the traditional track, including the Mathematical Sciences Major, Actuarial Science Major, and the Honors Program. There are about 45 students per year graduating in the traditional major, most of whom intend to become school mathematics teachers. Mathematical Sciences is a demanding major with breadth requirements in mathematics, statistics, computer science, and a related field in which quantitative methods are applied. Currently there are on average 5 students per year graduating in this option. The Actuarial Science Program currently graduates about 15 students per year. Almost without exception they find immediate employment in the insurance and banking industries. The Honors Program is designed to attract, cultivate, and challenge talented, highly motivated students. Students pursue a rigorous program of honors courses, taught in small classes, and typically complete their degree with a selection of graduate courses. There are about 10 honors students graduating each year, most of whom pursue graduate work in mathematics.

Service and Outreach. The Department is engaged in a number of service and outreach programs including the

- Mathematics and Statistics Learning Center (MSLC) — which provides tutorial and learning support services to Ohio State undergraduates, regardless of majors
- Ross Summer Mathematics Program — an intensive course for talented high school students
- Calculus Remote at the Ohio State University (CROSU) — a distance learning project, combining technology and the Socratic method of teaching, for high school students without access to classroom calculus
- Early Math Placement Test (EMPT) — an intervention program designed to help high school students make decisions about taking high school mathematics courses before enrolling in college
- Math 2001 Colloquium — a forum for discussing the wider issues facing the mathematical community.

Social life. The Department takes several steps to help newcomers (faculty, postdocs, and graduate students alike) to get a sense of the Department as a whole, to meet and become acquainted with their colleagues, and to exploit the opportunities available in our large active Department.

To help create social occasions for faculty, staff, students, and visitors to meet during the regular academic year, the Department holds a daily tea in the seventh floor lounge. At the start of Summer Quarter, when new graduate students arrive on campus, there is a reception to which all current graduate students and faculty are invited. The annual faculty reception is held in the fall, just after

classes begin, to introduce and welcome new faculty and postdocs to the Department. In a revival of an old tradition, over several weekends during Winter and Spring Quarters, the Chair and Vice Chairs invite small groups of faculty, postdocs, and their spouses and children to afternoon tea in their homes.

2.2 Critical Assessment and Proposed Actions

Critical Assessment. The size of the Department is a source of great strength. Almost every area of mathematics is represented among the research interests of the faculty, and this helps create a very stimulating intellectual environment. Advanced courses, topics courses, and seminars in a wide range of mathematical specialties are offered on a regular basis giving faculty, graduate students, and visitors ample opportunity to interact and learn from one another.

Unfortunately, however, the Department's size and complexity also has a downside. It can at times be difficult for new graduating students, postdocs, and faculty to fully appreciate the range of the Department's activities and to exploit all the opportunities available in the Department. It takes some time to become fully acquainted with all of one's new colleagues which can be a barrier to full participation in the teaching, research, and social life of the Department.

The postdoctoral program is quite successful. The terms and conditions of Zassenhaus Assistant Professors (ZAPs) are competitive, and their record of academic placement on leaving Ohio State is good. While the terms and conditions of Ross Assistant Professors (RAPs) are not as attractive as those of ZAPs, we have been very pleased with the quality of the first several classes of RAPs, and the initial data on their post Ohio State placement is encouraging. (Cf. Appendix (1)).

Several components of the undergraduate and outreach programs are particularly successful including Honors, Actuarial Science, and the Ross Summer Mathematics Program. These programs already receive a lot of faculty attention. The Undergraduate Committee has a standing charge to review undergraduate math course offerings and, in particular, service courses. In the last three years we have begun offering a reform calculus course (in addition to the traditional sequences) and a differential equations course which is team taught by mathematics and engineering faculty. In addition, we began to implement curriculum changes in courses for elementary school teachers in response to changes in the state licensure requirements. In the coming year, the Undergraduate Committee will undertake a review of the mathematics major with attention paid to issues such as modification and improvement of existing tracks, the possible creation of new tracks, advising of majors, and implementation of assessment mechanisms.

A clear weakness of the graduate program is the average time it takes to complete the PhD (cf. Appendix(1)). While there have been some very good academic and industrial job placements of our recent PhDs, there is clearly room for improvement. The Department has begun a systematic review of the graduate program. In the previous academic year, the Graduate Studies Committee made recommendations to improve the structure of advising and mentoring, and to improve the computer environment for graduate students. It also made recommendations to consolidate the number, frequency, and timing of graduate courses, in response to the smaller size of the program. The Department accepted and acted on these recommendations. In the coming academic year the Graduate Studies Committee will undertake a review of the breadth requirements, the system of

qualifying exams, and the distribution of credit hours required by students in the research and dissertation writing stage. Further steps in the revision are contingent upon the outcome of next year's work.

Proposed Actions. As a result of the above assessment, we have focused on devising new programs which connect graduate students to the faculty, postdocs, and their research interests much earlier and more systematically than in the past. This is the intended role of the *Invitation to Research* course and the system of *Working Group Rotations* which are described in greater detail in sections 2.3 and 2.4, respectively. Roughly speaking, the goal of Invitation to Research is to give beginning graduate students a bird's eye view of mathematics and to introduce them early on to a substantial fraction of the faculty. Working Groups are the main vehicle for vertical integration of undergraduates, graduates, postdocs, and faculty. They will, in particular, provide students with several hands-on experiences with mathematics during their second year.

Enhanced advising and mentoring of graduate students and postdocs will be undertaken (cf section 2.5). We will take steps to enhance the preparation of graduate students and postdocs through several professional development activities as outlined in section 2.6. Improvements to the undergraduate research experience are detailed in section 2.7. In that section, we also propose several pipeline programs which are intended to increase the early, positive experience in mathematics of undergraduates and to overcome barriers to graduate training.

2.3 Invitation to Research[NEW]

The advantage of a large department such as ours is the breadth of faculty research interests. However, this diversity of interests presents a difficulty to our beginning graduate students who have to make choices of research direction and course work based on limited prior knowledge. In order to expose our students to the research areas represented in the department early in their graduate career, we will offer the Invitation to Research lecture series.

Invitation to Research is a year long course with two class meetings per week. It will be taught in a very informal way in our lounge on Mondays and Wednesdays after the daily tea. The course is arranged as a sequence of modules of one to two weeks duration taught by various faculty and VIGRE postdocs. The entire sequence of modules will roughly span the research areas represented in the department. These areas are broadly divided into the following five groups, the division being neither exclusive nor exhaustive: *Algebra*, *Analysis*, *Applied Mathematics*, *Discrete Mathematics*, and *Geometry and Topology*.

In Autumn Quarter, Invitation to Research will consist of five modules, one for each of the above areas, taught by five senior faculty members. In subsequent quarters, these areas will be revisited several times by other faculty and postdocs. Instructors will connect their modules with the relevant Working Groups (described in the next section) and with relevant sequences of courses. The material covered in each Invitation to Research module will be written up and disseminated on the web.

The focus of each module will be on historical development, motivating ideas, examples, typical problems, and connections to other areas (intra-mathematical or interdisciplinary). Of course, given the limited time available (2-5 class hours), this will have to be done on a somewhat superficial

level. Nevertheless, we believe that it is possible to convey to students the general questions and ideas that motivate mathematicians in each particular area. Each module will be taught at the level of senior undergraduates; in particular, no advanced knowledge of graduate level mathematics is assumed or expected. Invitation to Research is aimed at first-year graduate students, but we will also encourage seniors majoring in mathematics, in particular our honors students, to attend.

Below is a partial list of faculty who have prepared modules for Invitation to Research. More detailed descriptions of these modules are available on the interim VIGRE website located at `http://www.math.ohio-state.edu/vigre`.

- Gregory Baker: *Scientific Computation*
- Vitaly Bergelson: *Ergodic Theory: From Statistical Mechanics to Modern Number Theory*
- Dan Burghelea: *Differential Topology*
- Timothy Carlson: *Mathematical Logic*
- Ruth Charney: *Introduction to Geometric Group Theory*
- Alexander Dynin: *Mathematical Physics: Three Quantum Revolutions*
- Yuval Flicker: *Modular Forms*
- Avner Friedman: *Partial Differential Equations*
- Yuji Kodama: *Integrable Systems*
- Peter March: *Probability and Stochastic Equations*
- Jeffery McNeal: *Several Complex Variables*
- Henri Moscovici: *Noncommutative Geometry*
- Andras Nemethi: *Geometry of Polynomial Maps*
- Neil Robertson: *Structural Graph Theory*
- Warren Sinnott: *Number Theory*
- David Terman: *Mathematical Biology*

To summarize, the primary goal of the Invitation to Research course is to provide first-year graduate students with a glimpse of the basic ideas, motivating examples, and research directions in a number of different areas of mathematics. This will give them a sense of what research is "all about" and provide them with a context for their graduate studies. Secondary goals of Invitation to Research are to introduce new students to a substantial fraction of the faculty and thereby to facilitate advising and mentoring activity. Furthermore, we believe that Invitation to Research can stimulate increased interaction among undergraduate and graduate students, postdocs, and faculty through active participation in the lecture series.

We end this section by giving two examples of Invitation to Research modules:

Ergodic Theory: From Statistical Mechanics to Modern Number Theory (Vitaly Bergelson)

While the roots of ergodic theory are in the kinetic theory of gases and celestial mechanics, modern ergodic theory stands at the junction of many areas. The goal of this module is to acquaint the students with some of the promising directions of research by focusing the discussion on three central topics: *recurrence*, *equidistribution* and *independence*. In the first lecture we shall give an overview of the historical roots of ergodic theory, discuss the notion of a *dynamical system* and consider some instructive examples: billiards, unimodal maps, continued-fraction transformations, automorphisms of compact groups and others. The main theme of the second lecture is recurrence. We shall discuss various examples from the theory of Diophantine approxi-

mations, celestial mechanics and Ramsey theory, and interpret them as recurrence theorems. We then try to formulate reasonable new conjectures. The third lecture will deal with the concept of independence. We will discuss the connections of ergodic theory with probability theory and devote some time to chaotic dynamics. The fourth lecture will be devoted to equidistribution in its broadest sense. The idea of invariant measures will be stressed, various convergence theorems will be discussed, and connections with and applications to statistical mechanics, number theory, and combinatorics will be pointed out. Throughout these lectures we will constantly emphasize the ubiquity of dynamical ideas in diverse, seemingly static situations.

Noncommutative Geometry (Henri Moscovici)

The study of certain "spaces" that arise in mathematics and physics forces us to adapt our geometric thinking to situations where the coordinates on the space do not commute. This has led to a novel geometric approach, based on a quantum perception of space, generalizing but distinct from the traditional description of a continuum filled with points (cf. Alain Connes's book *Noncommutative Geometry*). While the theory may seem strange and unfamiliar at first glance, its spirit and methods can be illustrated by relatively simple examples. This series of lectures will begin with Connes's charming proof of one of the most surprising results in synthetic geometry: Morley's theorem asserting that the trisectors of the angles of any triangle meet at the vertices of an equilateral triangle. When cleverly restated, as a statement involving three elements of the group of affine motions of the line, it can be verified by a direct computation. More remarkably, it makes sense over an arbitrary field (of characteristic different from 2). Rephrasing geometric properties in algebraic terms, without having to invoke commutativity, is only the beginning of the story. Deeper features of the theory, such as the natural occurrence of irrational dimensions, will be discussed next by means of a simple but highly nontrivial example: the noncommutative torus. This basic model of a noncommutative manifold can easily be introduced in terms of a simple differential equation describing the well-known Kronecker foliation. The fruitfulness of this study is exhibited even outside mathematics, by spectacular applications to the Quantum Hall Effect and more recently to string and M-theories. Finally, the inherently quantum nature of the appropriate notion of symmetry in noncommutative geometry will be demonstrated by discussing Kreimer's Hopf algebra of rooted trees and by illustrating the way it encodes the combinatorics of Feynman graphs in perturbative renormalization—to date the most successful technique for computing physical quantities in quantum field theory.

2.4 Working Group Rotations[NEW]

Working Groups will be the main vehicle for achieving vertical integration at Ohio State. Working Groups are units headed by faculty and postdocs united by a common research interest. They will include several faculty members and postdocs, and all interested graduate and undergraduate students will be encouraged to participate. Each VIGRE Postdoctoral Fellow will be required to join a Working Group, and they will be especially encouraged to form groups on their own in conjunction with other postdocs and faculty. Advanced graduate students will join the Working Groups of their advisor.

During their second year, VIGRE graduate trainees will rotate through three distinct Working Groups, one per quarter. During each quarter, students will work on a small project, perhaps individually or in small groups, including, possibly, undergraduate students. While their experience in the Invitation to Research course will be passive, students will now actively participate and engage in hands-on research. They will be exposed to a variety of research areas and to a relatively

large number of faculty, postdocs, and advanced graduate students.

We will also encourage, and actively recruit, our undergraduate students to participate in Working Groups, funded as VIGRE undergraduate trainees during the year or in the summer.

Student projects may vary considerably from reading an introductory paper or a survey paper, to exploratory computer simulations, to working on special cases and examples. At the end of the quarter, the graduate students will be asked to write up a project report and give an oral presentation. They will submit their report as a paper to an internal electronic VIGRE Working Group Journal, where it will be refereed and published. VIGRE postdocs will serve on the editorial board and act as referees. This will serve the dual purpose of giving both graduate students and postdocs important experience in communicating mathematics and developing professional skills.

Criteria for forming a Working Group will be deliberately kept minimal so as not to constrain the imagination and interests of the faculty. A Working Group must involve at least two faculty members; be willing to take on at least one graduate student and one undergraduate and to provide them with suitable projects; meet at least once a week; and last at least one quarter. It is expected that Working Groups will come and go as faculty interests and alignments change. We anticipate that some groups will be rather stable and continue on perhaps for years. Others may be shorter lived, for example, if the scope of the group is limited to a precise goal. Some groups will be fairly large and have the capacity to take on many students. Others may be smaller with one or two students. A given participant may belong to more than one Working Group. Overall, we expect that there will be a variety of Working Groups that is fully reflective of the diverse research interests of the faculty and that the total capacity to take on students will exceed the demands of the VIGRE program.

The weekly meetings will be used for various purposes. Postdocs and faculty members may give tutorials on the background required to work on the projects. There could also be informal discussions about the projects or progress reports given by students engaged in projects. We expect these meetings to improve communication skills and to break down barriers between undergraduates, graduate students, postdocs, and faculty.

Examples of Working Groups are given below with more detailed descriptions provided on the interim VIGRE website at `http://www.math.ohio-state.edu/vigre`.

- ALGORITHMS FOR GROUPS AND THEIR APPLICATIONS (*Peter Brooksbank*, *Koichiro Harada*, *Björn Sandstede*, *Ákos Seress*, *Ron Solomon*)
- AROUND SYMMETRY (*Dan Burghelea*, *Henri Moscovici*, *Steve Rallis*)
- BLOWUP OF SOLUTIONS TO PARTIAL DIFFERENTIAL EQUATIONS (*Avner Friedman*, *Yuan Lou*, *Saleh Tanveer*)
- COMPLEX GEOMETRY AND ANALYSIS (*Jeffery McNeal*, *Andras Nemethi*, *Fangyang Zheng*)
- COMPUTATIONAL NEUROSCIENCE (*Ed Overman*, *David Terman*, *Alice Yew*)
- DISTRIBUTION OF EXTREMA (*Peter March*, *Robin Pemantle*, *Boris Pittel*)
- JACOBI'S "FUNDAMENTA NOVA" AND SUMS OF SQUARES (*Soon-Yi Kang*, *Stephen Milne*, *Jaebum Sohn*)
- NUMBER THEORY (*Susan Goldstine*, *David Goss*, *John Hsia*, *Wenzhi Luo*, *Warren Sinnott*)
- READING CLASSICS (*Vitaly Bergelson*)

- SYMBOLIC DYNAMICS AND APPLICATIONS (*Vitaly Bergelson*, *Alexander Leibman*, *Björn Sandstede*)

A significant number of the Working Groups involve faculty from different areas: the first group in the above list, for instance, involves faculty working in algebra, group theory, combinatorics, and applied mathematics. We hope that Working Groups will therefore help to overcome some of the conventional divisions inside mathematics. Seeing faculty and postdocs discussing and struggling with unfamiliar concepts will be a positive experience for the participating undergraduate and graduate students.

Faculty members and postdocs will provide ideas for the projects well before the quarter commences. The descriptions of individual Working Groups will be distributed and advertised on the VIGRE webpage. Based on this information, students can then choose to participate in a group of their choice, subject to availability and the capacity of a group to take on students. Matching of students to Working Groups will be facilitated by designated members of the Coordinating Committee (cf. section 5).

To summarize, the primary goal of *Working Group Rotations* is to expose graduate students to research at an earlier stage, and in a more hands-on fashion, than previously. We expect that this closer working association with faculty will whet student's appetite for research and encourage them to attend colloquia and research seminars earlier in their career and with more confidence. It will also allow our undergraduate majors, in particular our honors students, to have an early glimpse of mathematical research.

We anticipate several desirable effects of these rotations.

- Participation in Working Groups will cause some circulation of graduate students among the faculty. This will help students to make a more informed choice of the area in which they want to do their thesis.
- Close working contact between graduate students, faculty, and postdocs will be established much earlier, leading to improved mentoring.
- Advanced graduate students may decide to rotate through additional Working Groups during their fourth and fifth year or decide to focus on participating in the activities of their advisor's Working Group. In either case, they will continue to be involved in the Working Group program during their graduate work.
- Postdocs will have the opportunity to advise undergraduates and beginning graduates in a supportive context where they can benefit from the experience without shouldering complete responsibility for their charges.

We end this section by giving more detailed descriptions of two Working Groups:

Computational Neuroscience (Ed Overman, David Terman, Alice Yew)

Complex spatio-temporal patterns of neuronal activity arise throughout the central nervous system. This activity has been observed in sensory processing, motor activities, and learning, and has been implicated in the generation of sleep rhythms, epilepsy, and Parkinsonian tremor. Basic questions involve how these rhythms are generated, how they are modulated, and what the behavioral correlates of the neuronal activities are.

A primary focus of this Working Group is to first develop mathematical models for cellular activity in a variety of neuronal systems and to then develop both analytic and computational tools for analyzing the complex patterns which emerge from the models. This will hopefully help experimentalists and theoreticians to better interpret data, create more sophisticated models, and design new experiments. Ongoing research projects include analysis of models for sleep rhythms, certain forms of epilepsy, scene segmentation, and Parkinsonian tremor. Members of this Working Group are also interested in other areas of mathematical biology including models for the cell cycle, cancer, genomics, and interacting species.

The synchronization of oscillations is an important issue in many biological systems. Graduate and undergraduate students will study small networks of model oscillators and use both mathematical and computational tools to find conditions under which the oscillators synchronize. They will then vary biological parameters of the model in order to determine how instabilities of the synchronous rhythm arise and what other sorts of stable rhythmic patterns may develop. The primary mathematical tools will involve dynamical systems methods including stability theory, bifurcation theory, and geometric singular perturbation theory.

Number Theory (Susan Goldstine, David Goss, John Hsia, Wenzhi Luo, Warren Sinnott)

Number theory provides many opportunities for vertical integration, since it can be entered at many different levels of background and experience. Many parts of it can be approached at a level quite accessible to advanced undergraduates and beginning graduate students and yet lead into areas suitable for doctoral thesis work and research. Below we give several examples of such topics and some references that approach these topics at various levels of sophistication.

p-adic Analysis: At its beginning levels, p-adic analysis provides a useful recapitulation of fundamental notions of real analysis in a novel setting. Direct connections to number theory can be made with the theory of p-adic L-functions, the beginning of which is quite elementary, and yet points to Iwasawa theory and "Main Conjectures" relating analytically defined p-adic L-functions with arithmetically defined characteristic polynomials.

Cryptology: The basics of cryptology, including the RSA and other public-key encryption methods, provide a suitable entry point. At a deeper level, one can study (and analyze the expected running times of) the various algorithms for factoring integers and for primality testing: some of these algorithms can be understood with very little background, while others (including the best algorithms now known) involve a deeper and more sophisticated knowledge of algebraic number theory: cyclotomic fields and elliptic curves. This is currently an active research area, and one in which new ideas may arise from cleverness, rather than extensive knowledge.

Coding Theory: As a complement to the *secure* transmission of data (cryptology) is the *reliable* transmission of data, the subject matter of coding theory. Again the beginnings of the subject are quite accessible, while the problem of finding efficient error-correcting codes leads directly into deeper parts of number theory: the theory of integral quadratic forms, with relations to lattices and sphere packing; the theory of algebraic curves, with connections to Goppa codes.

2.5 Advising and Mentoring

2.5.1 Advising and Mentoring of Graduate Students

Advising. Until a graduate student chooses a thesis advisor, he or she will typically be mentored by a member of the Graduate Advising Committee (GAC). The implementation of this committee

is a result of the deliberations of the Graduate Studies Committee during the 2000-2001 academic year (cf. section 2.2). The GAC will be a 6-8 member committee, chaired by the Vice Chair for Graduate Studies, having broad representation from the various research areas in the Department.

When new students accept an offer to come to Ohio State, they will be assigned by the Vice Chair to a member of the GAC. Students expressing a research interest will be assigned to an appropriate member. Others will be assigned according to the insight of the Vice Chair, who is intimately involved in the recruitment and admissions process. The relevant GAC member will be responsible for initiating contact with each of his assigned students before the end of Spring Quarter. An outcome of this contact will be an assessment of the student's needs and interests as well as an appointment for a face-to-face meeting sometime during the summer. Upon arrival on campus, students will then meet individually with the Vice Chair to discuss course options for Summer and Autumn Quarters.

The Vice Chair and GAC members will not only advise students—for example on academic and procedural matters—but also mentor them. Every aspect of the graduate student experience will be open for discussion including teaching, choice of research area, and choice of academic vs. industrial career. It is understood that the Vice Chair's initial assignment is preliminary and a student may change advisors at will. GAC advisors will meet with their students at least once per quarter, but more frequent meetings are encouraged. When a student chooses the thesis advisor, ideally by the beginning of the third year of graduate school, advising and mentoring responsibilities pass from the GAC to that faculty member.

We take several steps to encourage supportive interactions between new and advanced graduate students. Specifically: upon arrival in Columbus, new international students are shepherded around at first by current students from their country or region; there is a reception for new students at the beginning of summer and pizza parties in the Department and in faculty homes during the summer which bring together a wide variety of graduate students; and office assignments deliberately mix new and old graduate students.

Teaching. Teaching serves as an apprenticeship which fine tunes a student's communication skills, complementing his or her professional development as a mathematician. Accordingly, we take very seriously the training, supervision, and advising of our student teachers, and we provide many resources to ensure the quality of teaching and communication. To further motivate our TAs and to demonstrate our commitment to teaching, the Department recently introduced a Teaching Award that is awarded to three TAs each year.

Most graduate students at Ohio State are financially supported as Teaching Assistants. The Department employs three full time staff members for graduate student and TA support. Once the academic year begins, support for TAs takes on several forms. TA Support staff give each new TA feedback on teaching early in his or her first quarter of teaching. They offer individual consultation, classroom observation, videotaping, and critiques of teaching performance to each teaching assistant. Follow-up visits and individual consultations are available at the end of the quarter to assess each student's progress in teaching.

A one-hour weekly seminar for new TAs is provided, including discussion of classroom experiences and student issues. It is moderated by selected faculty and TA Support staff, with additional visits

by the faculty involved in the Summer Teaching Practicum (cf. section 2.6.1).

The Math 2001 Colloquium provides an additional forum for discussing issues related to teaching. As part of Ohio State's competitive Selective Investment Program that we were recently awarded, the Mathematics Department received funding to hire a senior faculty in mathematics education. We expect that this faculty will set up and organize a mathematics education seminar that will further strengthen our support and advising system for TAs and faculty.

2.5.2 Advising and Mentoring of Postdoctoral Fellows

Faculty take an active role in hiring postdocs by bringing promising candidates to the attention of the Advisory Committee, the committee which makes hiring recommendations to the Chair. This internal process by which hiring recommendations are made assures that each successful candidate has one or more supporters among the faculty. One of these supporters becomes the postdoc's assigned mentor. The mentor helps the new postdoc in making the transition to Ohio State and settling in. The mentor also works with the postdoc in new research collaborations and advises the postdoc on such diverse professional matters as drafting research manuscripts, choosing appropriate journals in which to publish, and in applying for external funding.

2.5.3 Advising and Mentoring of Undergraduate Students

Each undergraduate student has a faculty advisor. Advisor and advisee meet at least once per quarter to discuss the student's progress and course work. In addition, we employ three full time staff members as counselors whose role it is to give further advice on course work.

2.6 Professional Development

There are a number of issues crucial to the professional development of future mathematicians which should be addressed at appropriate times during the graduate traineeship. In section 2.6.1, we describe our existing teacher training course. This course will be supplemented by introducing a course in computer literacy (cf. section 2.6.2), a seminar on communicating mathematics (cf. section 2.6.3), and two industrial mathematics initiatives (cf. section 2.6.4) devoted to various aspects of professional development of graduate students.

2.6.1 Introduction to Teaching

Our current Introduction to Teaching course for all incoming graduate students is a practicum which lasts throughout the entire Summer Term. The teaching staff includes two faculty, an advanced graduate student with excellent teaching credentials, and the Department's TA Support staff. There are two course sections for teacher training—one for international and one for domestic students—which are combined for many of our activities.

All new students visit three actual classrooms taught by well-qualified, experienced TAs who have been awarded summer teaching support. This is a great opportunity for the students to see first-hand what good classroom practice is and to observe students in action. We also provide several opportunities for discussing case studies and role playing which cover many aspects of being a TA.

In addition, there are panel discussions with experienced TAs, discussions about pedagogy, student background and culture, and presentations about departmental policies and practices.

Students are given valuable experience presenting explanations of problems and concepts in a mock recitation classroom environment. Currently we have nine opportunities for this practice, for a total experience of 90-110 minutes per student. With each presentation, immediate feedback is given to the student, and a general discussion takes place with the entire group about effective pedagogy. Students are videotaped twice during the practicum, once early in the quarter in a recitation setting and once mid-quarter during a short lecture presentation. As each student is being videotaped, the entire class assesses the teaching performance, and a written summary of their assessment is provided to the student.

2.6.2 Computer Literacy[NEW]

This five week course is designed to familiarize our graduate students with the software they are most likely to encounter in a university setting. It will be taught in a computer laboratory with a minimum amount of lecturing by the instructor. The intent is to let the students experiment with the software, so that they learn by working on examples.

The course begins with an introduction to the operating systems Unix and Linux that are used in the Department. After learning the usage of the powerful editor Emacs, we focus on LaTeX 2_ε and TeX as the prevalent packages to produce manuscripts. We also introduce Postscript and PDF as formats for printing documents and disseminating them on the Web. This leads naturally to a primer in HTML and on tools that allow us to convert LaTeX 2_ε documents to HTML. We think it would also be useful to explain how literature searches of databases (such as the Citation Index or the Math Reviews) and on the Web are best done, and how documentation and answers to various computer and type-setting related questions can be found efficiently using the Web and newsgroups.

Afterwards, we focus on graphing tools such as Xfig and on various mathematical software packages including Matlab, Mathematica, and Maple. We also introduce XPPAUT which can be used to compute solutions to ODEs. If time permits, debuggers for Fortran, C, and C++ code will be introduced.

2.6.3 Communicating Mathematics[NEW]

The Communicating Mathematics seminar comes in two parts.

Part I will be taken by first-year graduate students in Spring Quarter. The students will learn how to give presentations of varying length and to different audiences, and how to write academic papers and reports. The goal of the seminar is to provide the students with guidelines, templates, and practice to accomplish the above tasks. We will visit colloquium and seminar talks, and discuss the presentations afterwards in detail. We will also discuss thoroughly a number of papers and reports. Graduate students will later practice and apply in various ways the theoretical knowledge acquired in the seminar. Working Group projects simulate, in a small way, the experience of writing an academic paper. Students will also be asked to give an oral presentation at the end of each *Working Group Rotation*. We will also ask students to write up short annual reports for summer fellowship

support and brief proposals for using the travel money provided by the Department.

Part II is aimed at postdocs and fifth-year graduate students. Its goals are to provide the participants with some of the skills that makes applying for positions and succeeding in an academic environment easier. We will learn what a successful application package should contain and how to build an attractive teaching portfolio. Through informal discussions and by reading sample papers, we will learn how a good paper should be structured and written, and how and where to submit papers. Other topics that will be discussed include how to write and react to referee reports. Perhaps one of the most difficult tasks for beginning postdocs is to decide on, and afterwards pursue, a research program and profile. This includes making decisions on what topics to work on, and where and how to apply for funding. These issues will be addressed through panel discussions with invited faculty.

2.6.4 Industrial Mathematics[NEW]

In order to broaden the experience of graduate and undergraduate trainees, to expand professional opportunities, and to open new career paths, we propose new initiatives in industrial mathematics. In the short term, we will strongly encourage our graduate students to take advantage of existing opportunities in industrial mathematics provided by MITACS. We intend to strengthen our own connections to industry by establishing an industrial mathematics seminar. We expect that with consistent effort we will build long term relationships with local companies resulting in internships and employment for our graduate students and postdocs.

MITACS Internship Opportunity. The Mathematics of Information Technology and Complex Systems (MITACS) is a Canadian Federal Network of centers of excellence and a joint initiative of Canada's three national mathematics institutes. Funded jointly by the federal government and more than 75 companies, MITACS encourages collaborative research partnerships between universities and emerging sectors of the economy: biomedical, commercial/industrial, information technology, manufacturing, and trade/finance. Currently, MITACS consists of 28 research projects involving 230 academics and more than 400 trainees (students and postdocs) located at sites across Canada.

Training of highly qualified personnel in critical areas of the economy where mathematical modeling can play a central role is a key objective of MITACS. As such, MITACS continually strives to identify new opportunities for such training. MITACS has agreed to provide opportunities for Ohio State graduate students in mathematics to compete for summer internships with MITACS sponsored projects. Students can apply to work on projects in any of the above sectors by submitting a letter of application and curriculum vita to the MITACS Head Office located at Simon Fraser University in Vancouver. MITACS will act as a clearing house to match these applications with project needs. Applications will be forwarded to appropriate project leaders who will choose successful applicants. Appropriate support funds will be determined by project leaders on a case by case basis and could include travel support, stipends, and/or living expenses.

Once chosen, graduate students will spend about three months working with both academic and industrial scientists on applied research problems. The Ohio State Department of Mathematics will encourage its graduate students to apply for MITACS internships. MITACS encourages applications

from Ohio State to compete for these positions.

Industrial Mathematics Seminar. Columbus is a regional banking and insurance center and has a growing high technology sector. We already have an active Actuarial Science seminar, mainly aimed at undergraduate majors, with speakers from banks and insurance companies. Guest speakers from industry have also made occasional visits to the Department in the past, particularly in the Applied Mathematics seminar.

We intend to increase these activities gradually over the next two years to the point where a separate industrial mathematics seminar can be up and running on a regular basis. Local companies and local affiliates of national and international companies which employ mathematically trained professionals will be identified. We then make contact with representatives of these companies with a view to arranging reciprocal visits: of a faculty member to the company and, subsequently, of the representative to campus to speak in an industrial mathematics seminar. We will follow up contacts with our recent PhD graduates who have secured employment in industry and are natural candidates for establishing such contacts. The expected benefits to companies will be access to a pool of potential employees possessing a high degree of mathematical background and training. The expected benefit to Ohio State is greater breadth of experience for students and an increased pool of potential employers of our graduates.

2.7 Pipeline Programs

The goal of the following collection of programs is to increase the number of students who have an early, positive mathematical experience. It is hoped that this will increase both the number of undergraduate math majors and the number of applicants to graduate school.

2.7.1 Ross Young Scholars Program

For 44 years the Ross Summer Mathematics Program has been a successful multi-level endeavor, an on-going model of vertical integration. In recent years the program has involved high school students, high school teachers, college undergraduates, graduate students, postdocs, and senior professors. Communication among all the participants has been encouraged in various ways, not only within the classrooms and working seminars, but also at meals and in the dormitories. One of the great successes of the Ross Program is the sense of community and continuity among the participants, across the various types of students and teachers.

The Ross Program centers on an intensive number theory course for talented high school students. For eight weeks each summer, students are immersed in a world of mathematical discovery. For most of them this is the first time they will be asked to consider unfamiliar questions, to develop methods that they have not seen before, and to justify every answer. The number theory course is organized around a series of daily problem sets. These sets invite students to contemplate a variety of seemingly simple questions about numbers and their relationships. As the summer progresses, students are encouraged to investigate these questions in increasing depth, and to return to them periodically as their skill at abstract reasoning and their collection of available tools become more powerful. This spiraling of concepts is summarized in the Ross Program's motto: *Think deeply of*

simple things.

In order for this intensely problem-based approach to succeed, the students must be given careful feedback on their work. This role is played by the counselors who live in the dormitories along with the younger participants. The counselors, who are alumni of the Ross Program currently studying as undergraduates at universities across the nation, work directly with the first-year students, contributing a tremendous amount of time and energy to the job. The counselors also participate in challenging advanced courses and individual independent studies.

2.7.2 Honors Program

The Honors Program is designed to allow talented, highly motivated undergraduates to fulfill their mathematics course requirements for the bachelor's degree by taking honors and graduate courses. Honors courses are small, consisting of perhaps 8-12 students and are taught by dedicated instructors. There are four main course sequences: an introduction to analysis taken by freshmen; linear algebra, differential equations and complex analysis for sophomores; abstract algebra; and in alternate years differential geometry or number theory. Honors students often finish their course work with graduate courses.

Honors students typically engage in a number of additional activities. One is Radical Pi, a math club organized by undergraduates which arranges an informal lecture series by speakers drawn from undergraduates, graduates, postdocs, and faculty. Another is the Rasor-Bareis-Gordon examination, a competitive exam organized each year by the Department in which undergraduates at all levels compete to solve challenging problems.

Our Honors Program has been very successful. Our honors students have regularly won prestigious Goldwater awards and often go on to graduate work in top research departments across the nation.

2.7.3 Undergraduate Research Experience[NEW]

We plan to draw together and build upon two components of the Department's activities to form the undergraduate research experience: the Ross Summer Mathematics Program and our Working Groups.

Working Groups. We are convinced that our advanced math majors, in particular our honors students, will find the Invitation to Research course and the hands-on experience provided by Working Groups quite attractive. As VIGRE undergraduate trainees, they will participate in Working Groups of their choice without the constraint of having to rotate through on a quarter by quarter basis. Accordingly, they will have the opportunity to interact with graduate students, faculty, and postdocs on a gentler time scale, more appropriate to their background and preparation.

Working Group activities may also take place during summer, giving also counselors of the Ross Program the opportunity to participate. In addition, we expect to be able to recruit alumni of the Ross Program and Ohio State students as VIGRE undergraduate trainees.

Ross Program. During summer, undergraduate math majors, particularly honors students, and beginning graduate students sometimes participate as first-year students in the Ross Program alongside the high school students. This is possible because the problem sets involve questions at many

levels, from purely numerical exercises leading to conjectures, to formal proofs of elementary statements about integers, to generalizations involving other number systems. Students with different backgrounds work at different paces and depths, all thinking about the same abstract ideas.

A second opportunity for undergraduate research experience will be through additional activities in connection with the Ross Program. We will enhance the Ross Program by presenting the counselors with further opportunities to pursue mathematical research in the form of projects. This is being implemented in the Cryptology course taught this summer by Alice Silverberg: each student is working on a project involving current research, to be written up as a paper and presented to the others in the class. Starting next year, projects will also be incorporated into the probability course for second-year students taught by Peter March. Once again, we expect to recruit alumni of the Ross Program and Ohio State students as VIGRE undergraduate trainees to participate in these projects.

2.7.4 National Undergraduate Research Conference[NEW]

There is an increasing number of programs around the country which afford undergraduates a chance to experience the thrill of mathematical discovery. Existing REU sites are good examples, as are the undergraduate research experience components of currently funded VIGRE programs.

To support these undergraduate researchers, we intend to organize each summer a national Undergraduate Research Conference which will bring a substantial number of promising young researchers from around the country to Ohio State to participate in a conference showcasing research projects by undergraduates. We anticipate two benefits from this endeavor. First, it allows young talented and motivated scholars to form a supportive network early in their mathematical careers. Second, it gives our honors students and participants of the Ross Program, both high school students and counselors, the opportunity to interact with conference participants from around the country.

The conference format will consist of a two day conference—Friday noon until Sunday noon—and will include featured speakers drawn from research universities, government agencies, and industrial laboratories. Between featured talks, there will be short talks and poster sessions by undergraduates which will provide an appropriate forum for the dissemination of the results of undergraduate research projects.

2.7.5 Overcoming Barriers to Entry to Graduate Work[NEW]

Each year, there is a group of students in our applicant pool who are well recommended and interested in graduate work but for various reasons have inadequate preparation. For example, we typically get several applications each year from students at small liberal arts colleges, where there may not be enough math majors to justify an upper division course in analysis or algebra. In the past we have felt it unwise to admit such students.

This year we have admitted a selection of these students whom we judge will be able to succeed in our graduate program if provided with suitable support. They will be carefully advised when they arrive in the summer, and their progress in our summer Headstart Program will be carefully monitored. In the fall, they will be enrolled in a seminar which will accompany our introductory graduate courses in analysis and algebra. This seminar will meet twice a week with two faculty

members, where these students can be helped on a personal basis with digesting new material and learning to write mathematics well.

We expect these students to be almost exclusively domestic students, and some will likely belong to traditionally underrepresented groups in mathematics. They will be supported as regular graduate TAs in the Department.

2.8 Trajectories

We now account for the proposed activities by group (additional activities can be found in section 3).

2.8.1 Undergraduate students

We will vertically integrate undergraduate students through research exposures and experiences that take place in the Working Groups during the entire year and as part of the Ross Program during summer. To bring undergraduates from different universities together to share their experiences, we intend to organize an annual Undergraduate Research Conference.

2.8.2 VIGRE graduate students

We envisage the following sample time table for VIGRE graduate students:

1st year:	Funding	NSF
	Summer	Computer Literacy Seminar
	VIGRE	Invitation to Research,
		Communicating Mathematics Seminar I
	Course work	PhD courses
	Examination	PhD Qualifying
2nd year:	Funding	NSF and TA
	Summer	Teaching Preparation Course
	VIGRE	Working Group Rotations
	Course work	PhD courses
	Teaching	Teaching experience as TA
3rd year:	Funding	Lecturer and TA
	VIGRE	Industrial Mathematics Internship (optional)
	Course work	Finish PhD courses
	Teaching	Teaching experience as TA and independent lecturer
	Research	Select PhD advisor
	Examination	PhD Generals
4th year:	Funding	NSF and TA
	VIGRE	Participation and Rotation in Working Group
	Course work	Topics courses
	Research	Dissertation research
	Teaching	Teaching experience as TA or independent lecturer
5th year:	Funding	NSF
	VIGRE	Communicating Mathematics Seminar II
	Course work	Topics and Electives
	Research	Completion of dissertation
	Examination	PhD Defense

Besides starting their course work, entering VIGRE graduate students also attend the Invitation to Research lecture series in their first year. In their second year, they rotate through three different Working Groups, where they collaborate in an integrated fashion with undergraduates, other graduate students, postdocs, and faculty. They also engage in teaching as TAs with a reduced load. In the third year, graduate students finish their course work and choose their thesis advisor. It is during this third year that they get involved in teaching both as TAs and afterwards as independent lecturers (teaching typically one of the courses that they have previously been a TA for to ease the transition). The emphasis of the fourth and fifth year is dissertation research. During this time, graduate students participate in the Working Groups of their advisors and take advanced topics courses (we offer about 15 different topics courses each year). We would also strongly encourage our students to apply for internships through MITACS. Part of the fifth year will be devoted to preparations for the time after graduation: if a graduate student decides to pursue a career in academia, they together with their advisors would work on research and teaching portfolios, writing papers, and preparing job applications; if the decision is in favor of a career in industry, we would advise and mentor students to possibly pursue an M.Sc. in Statistics or Computer Science, or to take our courses in actuarial science or financial mathematics.

2.8.3 VIGRE postdoctoral fellows

VIGRE postdocs as well as ZAPs engage in various activities. We will encourage VIGRE postdocs to teach some of the two or four-hour lectures in the Invitation to Research series. Each VIGRE postdoc will participate in at least one of the Working Groups: this could be the Working Group of the postdoc's mentor, but every postdoc is free to choose a different Working Group or to organize a new group together with other postdocs or faculty. To give an example, several ZAPs and tenure track faculty recently organized a working seminar on complex dynamics that was also attended by graduate students. Through the Working Groups, postdocs work with other faculty, graduate students, and undergraduates. A significant number of RAPs and ZAPs is involved in the Ross Program, and we certainly hope that this tradition continues with VIGRE fellows.

The reduced teaching load of VIGRE fellows notwithstanding, we hope that teaching will be a major positive experience for VIGRE postdocs. We expect that postdocs teach a variety of course on all levels, including calculus classes, upper-division undergraduate courses, as well as introductory, advanced, and topics graduate classes.

Part II of the Communicating Mathematics Seminar is aimed at postdocs as well as advanced graduate students. VIGRE postdocs will also serve as editors and referees in our electronic journal where graduate students publish their Working Group reports. Through our mentors and advisors, we will encourage postdocs to submit grant proposals to appropriate federal agencies.

3 Description of the Optional Components

The Ross Summer Mathematics Program is an excellent example of how vertical integration of mathematics students, teachers, and researchers can be pursued in a deep and meaningful way. By considering simply stated questions from several directions and depths, students attain an understanding of how professional mathematicians and scientists work: gathering data, looking for patterns and analogies, making conjectures, and finally testing and proving those conjectures.

In the Ross Program for 2001 we began a new component for high school teachers. The Ross Program has involved teachers several times in the past. In fact, Arnold Ross began a teacher training program at Notre Dame in 1947 that eventually evolved into a program for students by 1960. That program moved to Ohio State in 1964 and has now become the Ross Summer Mathematics Program.

The current version of the teacher component began in 2001 with a start-up grant from the Park City Mathematics Institute. During this first summer we had five teacher participants attending for 3 weeks. Those teachers were enthusiastic about the "exploration method" used in the Ross Program. They were eager to try some of those pedagogical techniques in the classes they teach. They supported the idea of having occasional meetings during the academic year. Those meetings would serve as forums for those teachers to explain how they applied the Ross techniques in their own classes, and as a platform for recruiting some of their colleagues to have the Ross experience next summer. We plan to organize three such short meetings per year.

The teacher participants are involved in the program for the first 3 weeks, spending 4 hours in classes each day. They begin each day by attending the morning lecture on number theory along with the young students. The following two-hour workshop and a one-hour seminar focus on the number theory problems and various approaches and strategies to solve them. During the workshop and seminar times they work interactively on the assignments, discussing the ideas with one another with some guidance by the instructors. The crucial ingredient is the formation of a culture of exploration among the participants.

This teacher component of the Ross Program is an excellent outreach opportunity for the Department. It could have a real positive impact on the local schools, and will improve the articulation between the University and local high schools. We expect that this program will help improve the standing of the Department within the local math education community.

We propose to expand the teacher program to a class of 15 teachers. Such an extension is rather expensive since the teachers require stipends and since a low ratio of instructors to participants is needed for the exploration method to work effectively. In addition, we plan to organize three meetings per year for high school teachers during the academic year.

4 Recruitment and Retention

4.1 Recruitment and Retention of Graduate Students

Advertisement. Ohio State's graduate program brochure, containing a brief description of the graduate program, contact information, and the anticipated nine month TA stipend, is distributed nationally and internationally to a large number of colleges and universities. We also advertise our graduate program on the departmental webpage.

Faculty Contacts. The Vice Chair for Graduate Studies maintains contacts with corresponding faculty at other institutions around the country regarding stipend levels, interest among their students in pursuing graduate work, etc., and provides such information in return. Faculty are encouraged to solicit applications from students of their colleagues in other departments of mathematics. The Vice Chair, occasionally accompanied by another faculty member or advanced graduate student, regularly visits colleges and universities throughout the state to recruit students. Students to whom we have made offers are invited to visit campus. After these visits, the Vice Chair, members of the Graduate Recruitment Committee, and selected faculty make follow-up contacts by telephone or email to encourage those students to study at Ohio State.

Visitation Days. During Winter Quarter, the Department organizes one or more Visitation Days which bring to campus student to whom we have made offers. Students arrive Friday and leave Sunday. On Friday the students walk through campus and department facilities, visit a graduate course, and meet with faculty and students over refreshments. Friday evening there is a reception at the Chair's home. Saturday afternoon, students are shown around Columbus by a number of graduate student volunteers who also take them out in the evening to sample the nightlife. We find this even split of formal interactions with faculty and informal interactions with current Ohio State students to be an effective recruitment device. In arranging Visitations Days, care is taken to involve a mix of faculty that includes women, researchers from different areas, and faculty originating both from the United States and abroad.

Recruitment of Traditionally Underrepresented Groups. The Graduate Program Student Visitation Day, organized with the help of the Ohio State University Office of Minority Affairs, is an annual program which identifies minority students interested in graduate work in mathematics and brings them to the Department to visit with faculty and students. To increase our minority recruiting effort we will initiate contacts with minority professional organizations around the nation and historically black colleges within the state. We hope to build upon these contacts to the point that they develop into relationships with Ohio State that bring greater numbers of minority students to our graduate program.

Two of the five hires made at Ohio State in the last two years were women. We hope and expect that greater numbers of women graduate students will be recruited to Ohio State, and be retained in greater numbers, as a consequence of the presence of excellent women mathematicians on the faculty.

Competitiveness. Ohio State offers each student accepted to the program a fellowship in the summer prior to their first year of graduate studies. This allows students to settle into Columbus, take the teacher preparation course and problem solving courses, make new acquaintances, and get

oriented to graduate student life before Autumn Quarter begins. We offer competitive TA stipends, good computer support, and high quality physical space for graduate student offices.

Advising and Mentoring. The Vice Chair for Graduate Studies and the Graduate Advising Committee provide mentoring and advice to all graduate students. To help new students become acquainted with faculty research interests, the Department compiles and distributes a faculty survey handbook. The handbook gives detailed information about each faculty's current research, current advisees, availability to take on new students, and recommended course work to prepare for research in his or her area. Each year every graduate student participates in a thorough and thoughtful assessment of his or her progress.

Ongoing Improvements to the Graduate Program. Over the last several years, the Department has introduced several new courses including a sequence in financial mathematics (Boris Mityagin), a sequence in mathematical modelling (Ed Overman) and a course in cryptology (Alice Silverberg). Other recent improvements and recommendations have been described in section 2.2.

We believe that the principal components of this VIGRE proposal, especially the *Invitation to Research* and *Working Group Rotations*, will be very helpful in the recruitment and retention of graduate students.

4.2 Recruitment and Retention of Postdoctoral Fellows

We advertise nationally and internationally for Zassenhaus and Ross Assistant Professor positions. Faculty also take an active role in bringing promising candidates to the attention of the Advisory Committee, the committee which makes hiring recommendations to the Chair. The internal process by which hiring recommendations are made assures that each candidate has one or more supporters among the faculty. In the past, this has been very helpful in recruitment as well as retention. As is the case with graduate students, we believe that the primary components of the VIGRE proposal, especially the *Invitation to Research* and *Working Group Rotations*, will be very helpful in recruitment and retention of strong postdoctoral fellows.

4.3 Recruitment and Retention of Undergraduates

We will recruit VIGRE undergraduates among our math majors, especially among honors students, and through the network of counselors of the Ross Summer Mathematics Program. The Radical Pi undergraduate math club will play an important role in advertising opportunities for undergraduate research experiences for students at Ohio State. The network of alumni of the Ross Program will be another source of undergraduate trainees. VIGRE opportunities will be advertised on the Department and Ross Program web pages.

5 Organization and Management Plan

Coordinating Committee. The organization and management of the project rests with the VIGRE Coordinating Committee. This committee consists of seven faculty, one staff, and two VIGRE participants:

- Peter March (PI and Department Chair)
- Vitaly Bergelson, Henri Moscovici, Björn Sandstede, Daniel Shapiro, Warren Sinnott (Co-PIs)
- Yung-Chen Lu (Vice Chair for Graduate Studies)
- Cindy Bernlohr (TA Support staff)
- VIGRE graduate trainee representative
- VIGRE postdoctoral fellow representative.

Peter March will chair the committee and will initially appoint the graduate and postdoc representatives. Thus, the broad research interests of the faculty are represented on the Coordinating Committee as is the departmental administration and staff.

Faculty on the Coordinating Committee have overall responsibility for the primary components of the project:

Peter March	(Department Chair) ◦ Recruitment, retention, advising, and mentoring of VIGRE postdocs ◦ Pipeline Program: Overcoming the Barriers to Entry to Graduate Work ◦ Project performance evaluation
Vitaly Bergelson	Pipeline Programs: ◦ Recruitment and retention of VIGRE undergraduate trainees ◦ Undergraduate Research Experience
Yung-Chen Lu	(Vice Chair for Graduate Studies) Recruitment, retention, advising, and mentoring of VIGRE graduate trainees
Henri Moscovici	Invitation to Research
Björn Sandstede	Professional Development
Daniel Shapiro	(Vice Chair for Lower Division Studies) Pipeline Programs: ◦ National Undergraduate Research Conference ◦ Ross Program Teacher Component
Warren Sinnott	Working Group Rotations

Cindy Bernlohr's role is to advise the committee on graduate teaching issues. The graduate and postdoc representatives' role is to provide input to the committee from VIGRE project participants.

Organization. Next we describe some of the organizational details specific to each of the primary components. It is understood that while the Coordinating Committee members have management responsibility for particular components and may, in fact, carry out some of these tasks, some tasks will be delegated to other faculty. Peter March is Department Chair, PI of the VIGRE

Project, and Chair of the VIGRE Coordinating Committee. He will ensure that as the primary components come online and are made to articulate with one another, appropriate faculty will undertake appropriately delegated tasks.

Invitation to Research:

- solicit and encourage faculty and postdocs to prepare modules and lecture notes
- coordinate the schedule of lectures
- ensure appropriate rotation of lectures through the five main research areas mentioned in section 2.3
- work with lecturers to ensure that their talks are delivered at a pace and level suitable to the aims of the series and the background of the intended audience.

Working Group Rotations:

- solicit and encourage faculty and postdocs to form Working Groups
- facilitate the quarterly matching of graduate students and Working Groups
- serve as editor in chief of the VIGRE Working Group Journal
- coordinate with representatives of Working Groups to ensure they meet regularly and provide appropriate student projects
- coordinate with representatives of Working Groups to ensure that student presentations and reports are completed in a timely fashion.

Advising, Mentoring, Recruitment, and Retention:

- our advising, mentoring, recruitment and retention activities and responsibilities, and their implementation, have been outlined in previous sections:
  - see section 4 for recruitment and retention
  - see section 2.5 for advising and mentoring of undergraduate students, graduate students, and postdocs
- personnel involved in and responsible for advising and mentoring:
  - the Graduate Advising Committee currently consists of Gregory Baker, Timothy Carlson, Andrzej Derdzinski, Neil Falkner, Zbigniew Fiedorowicz, David Goss, and John Hsia
  - the designated Coordinating Committee members (Peter March, Vitaly Bergelson, Yung-Chen Lu) will consult regularly with faculty mentors to ensure that sufficient contact between mentors and VIGRE participants (graduate trainees and postdoctoral fellows) is maintained.

Professional Development:

- Introduction to Teaching:

  Two faculty members, Phil Huneke and Yung-Chen Lu, and one staff member, Cindy Bernlohr, currently coordinate this activity
- Computer Literacy:

  Ed Overman has agreed to teach this course in Summer Quarter 2002
- Communicating Mathematics:

  Björn Sandstede has agreed to develop this course

- Industrial Mathematics:

  Ed Overman has agreed to begin development of the Industrial Mathematics Seminar. Yung-Chen Lu (Vice Chair of Graduate Studies) will coordinate MITACS applications (cf. section 2.6.4).

Pipeline Programs:

- Undergraduate Research Experience
  - coordinate recruitment of VIGRE undergraduate trainees
  - coordinate matching of undergraduates with the Ross Program or a Working Group
- National Undergraduate Research Conference
  - initiate and maintain contacts with national summer mathematics programs, REU sites, appropriate faculty at VIGRE sites, and national professional associations including AWM, MAA, NAM.
  - organize local arrangements, advertising, and scheduling
- Overcoming Barriers to Entry to Graduate Work
  - coordinate with Vice Chair for Graduate Studies, Graduate Recruitment Committee and Graduate Advising Committee to identify beginning students who will benefit from this support mechanism
  - John Hsia and Peter March have agreed to develop the seminar.

Commitment. The primary components of this proposal have the widespread support of the faculty.

Sixteen senior faculty have already volunteered to deliver Invitation to Research modules in one of the five main research areas. Even before soliciting the involvement of junior faculty and postdocs, there are enough modules for a complete year.

There are already ten proposed Working Groups involving 27 faculty (20 Professors, 4 Associate Professors, 3 Assistant Professors) and four postdocs (three Ross and one Zassenhaus Assistant Professor). We anticipate that in equilibrium there will be 6 VIGRE graduate trainees, 4 students on university fellowships, perhaps 10 regularly supported TAs and 10 VIGRE undergraduate trainees involved in Working Group Rotations. Using these figures, there is on average two graduate students and one undergraduate student per Working Group per quarter. Thus even before soliciting Working Groups from among the junior faculty and postdocs, there is enough capacity in these Working Groups to accommodate the demands of the program.

In all 33 professors, comprising nearly one half the faculty, are committed to Invitation to Research modules and Working Group Rotations. We expect this number to rise as the primary components of the project come online.

6 Performance Assessment

6.1 Goals, objectives, and indicators

The goals of this VIGRE proposal are to broaden the preparation of undergraduates, graduate students, and postdoctoral fellows in the mathematical sciences, and to improve education activities which stimulate teaching, learning, and research interactions across conventional boundaries of academic age, rank, and discipline.

We believe that the primary components of this VIGRE proposal, as described in section 2, will allow us to attain these goals. We have several objectives in mind, including:

- increased research experience opportunities for undergraduates
- greater pre-dissertation contact between graduate students and faculty
- improved preparation of graduate students and postdocs for academic and non-academic employment
- broader exposure of postdoctoral fellows to faculty roles and responsibilities
- improved advising, mentoring, and support.

6.2 Assessment and evaluation of VIGRE graduate trainees

For each VIGRE graduate trainee, the following specific indicators will be monitored:

- performance in courses
- timely passage of benchmarks such as breadth requirements, qualifying and general exams, and the foreign language requirement
- timely progress in dissertation research
- teaching evaluations including student, peer, and self-evaluations
- degree and quality of participation in principal components of the VIGRE program

Assessment. All graduate students are reviewed once a year to gauge academic performance, to estimate time-to-degree, and to assess teaching performance. There are a number of formal academic requirements that a student must satisfy before beginning work on a dissertation: the qualifying exams, the breadth requirements, the foreign language requirement, and the general exam. The student's progress through these formal requirements is summarized on the annual academic survey, distributed to each student in February, which the student and his or her advisor review and update as necessary. The advisor must sign the survey form before it is returned.

For students who have passed the general exam, an additional component of assessment is required, which is appropriate to the less structured academic environment of students in the research stage. (This will be considered for approval by the Graduate Studies Committee in the 2001-2002 academic year). Advanced graduate students are asked to compose a 1-2 page progress report to be shared with their thesis advisor by the middle of Winter Quarter. The report is an opportunity to review teaching, research, and course work and to set goals for the following year. The student, the thesis advisor, and a second faculty member appointed by the Vice Chair for Graduate Studies in consultation with the advisor will then meet to hear the student's presentation of the progress report. The second faculty member serves as an auditor whose role is to listen and ask questions.

The presence of the auditor creates the occasion to thoughtfully discuss the student's progress in all areas. After the meeting, the thesis advisor will append comments to the student's report which will then be included in the annual evaluation.

Evaluation. Annual evaluation of all our graduate students is made by the Graduate Studies Committee. Good academic standing according to university standards and timely academic progress according to departmental standards are considered. If the student is supported as a TA, satisfactory teaching performance as gauged by student evaluations and faculty/staff observations and responsible discharge of duties are considered as well.

Letters of renewal (or non-renewal) of support are given to the students before the end of the academic year. There is ample time before year end for the Graduate Studies Committee to consider any appeals.

6.3 Assessment of postdoctoral fellows

The following specific indicators will be monitored for each VIGRE postdoc:

- number of papers written, submitted, or accepted for publication; and in which journals
- external funding of research
- number of invited talks given
- number of local seminar talks
- degree and quality of participation in principal components of the VIGRE program
- teaching evaluations including student, faculty, peer, and self-evaluations.

As with VIGRE graduate trainees, during the Spring Quarter of each year, VIGRE postdocs will draft a 1-2 page progress report which addresses all areas of performance including teaching, research, and participation in principal components of the VIGRE program. The Department Chair, or a designated member of the Coordinating Committee, will serve as auditor in an annual meeting with the VIGRE postdoc and his or her mentor. After the meeting, the mentor will append comments to the report, and a copy will be given to the Coordinating Committee. The annual assessment of postdocs will take the form of an evaluation letter from the Department Chair. The Chair will draft the letter using as a basis the progress report and the indicators listed above as well as input solicited from the mentor and other faculty.

6.4 Assessment of VIGRE undergraduates

Each undergraduate will be asked to draft a brief summary report of their activities at the end of each period of VIGRE support. The summary will be discussed with the student's advisor, and the advisor will append comments, if appropriate, and sign the report.

6.5 Assessment of VIGRE teacher participants

Similar to assessment of VIGRE undergraduates.

6.6 Performance evaluation plan

The following specific indicators will be monitored for the duration of the VIGRE project:

- pre-VIGRE qualifications of each participant
- post-VIGRE placement (as appropriate) of each participant
- number of undergraduate participants
- number of teacher participants
- number of women participants (undergraduates, teachers, graduate students, postdocs)
- number of minority participants (undergraduates, teachers, graduate students, postdocs)
- retention of graduate trainees
- time-to-degree
- number of industrial internships
- self-reported impact of participant's involvement in the VIGRE program on career or career path
- information from follow-up contact 1-2 years after participation in the VIGRE project.

These data, and the accumulated annual assessments of VIGRE participants described above, will form the basis of the performance assessment. We will convene an external review panel consisting of three distinguished mathematicians to conduct a site visit of the Ohio State VIGRE project in the Spring Quarter of Years 1, 2, and 4 of the project. The Review Panel will be provided with the above data and materials prior to their visit and will have such access to the facilities and participants as they find necessary. The Review Panel will be asked to draft a report evaluating the performance of the VIGRE project on the basis of the stated goals and objectives. They will also be asked for constructive criticism.

The VIGRE Coordinating Committee will include a summary of the data, the Review Panel report, and intended changes and improvements to the VIGRE project as part of its annual report to the NSF.