

## REVIEW PROBLEMS FOR EXAM 2

(1) Solve the following initial value problems:

$$y'' + 2y' + y = 0, \quad y(0) = 2, \quad y'(0) = -2,$$

$$y'' - 4y' + 5y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

and

$$y'' + 5y' + 3y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

(2) Consider the differential equation

$$y'' - y' - 6y = 5 \sin t.$$

(a) Find the general solution of the corresponding homogeneous equation.

(b) Find a solution  $Y$  of the nonhomogeneous equation by using the method of undetermined coefficients.

(c) Give the general solution of the nonhomogeneous equation.

(3) Find the general solution of the equation

$$y'' + 2y' - 3y = 1 + e^{2t}.$$

(4) Consider the differential equation

$$(1 - t)y'' + ty' - y = 2(t - 1)^2 e^{-t}, \quad 0 < t < 1.$$

Verify that  $y_1(t) = e^t$  and  $y_2(t) = t$  are solutions of the corresponding homogeneous differential equation. Use the variation of parameters method to find a particular solution of the nonhomogeneous differential equation.

(5) Consider the functions  $y_1(t) = e^t$  and  $y_2(t) = t$ . Compute the Wronskian of these functions. Are they linearly independent? Without solving the differential equation

$$y'' - t^2y + 3ty = 0$$

state why  $y_1(t)$  and  $y_2(t)$  cannot be solutions of the given differential equation.

(6) If  $y_1$  and  $y_2$  are linearly independent solutions of  $ty'' + 2y' + te^t y = 0$  and if  $W(y_1, y_2)(1) = 2$  find the value of  $W(y_1, y_2)(5)$ .

(7) Are the functions  $f(t) = \cos t$  and  $g(t) = \sin t$  linearly independent? Justify your answer.

(8) Use the solution  $y_1(t)$  given below to find a second solution of the differential equation

$$(t + 1)y'' - (t + 2)y' + y = 0, \quad t > -1, \quad y_1(t) = e^t.$$

(9) Solve the given differential equation by means of a power series about the given point  $x_0$ . Find the first two terms in each of two linearly independent solutions (unless the series terminates sooner).

$$2y'' + xy' + 3y = 0, \quad x_0 = 0$$

and

$$(1 - x)y'' + xy' - y = 0, \quad x_0 = 0.$$

(10) Homework-type applications from sections 3.8 and 3.9.