

MATH 649 HW 4

1. Show that A_E has a model with exactly one nonstandard element.
2. Suppose T is a satisfiable theory in \mathcal{L}_E such that $\#T$ is Σ -definable and T contains all true sentences in Σ .
 - a. Show that if A is a finite set of sentences such that $T \cup A$ is satisfiable then $Cn(T \cup A)$ is not a complete theory.
 - b. Find a function $s \mapsto A_s$ from $\{0, 1\}^*$ (the set of finite sequences of 0s and 1s) into finite sets of sentences of \mathcal{L}_E such that
 - $T \cup A_s$ is satisfiable for each s .
 - $A_s \subseteq A_t$ if t extends s .
 - For each s , there is a sentence σ_s such that $\sigma_s \in A_{s0}$ and $\neg\sigma_s \in A_{s1}$.
 - c. Use part b to show that T has 2^{\aleph_0} complete extensions by finding a 1-1 function from ${}^\omega 2$ (the set of functions from ω into $\{0, 1\}$) into the collection of complete theories extending T .
3. Prove Theorem 8.6.