

MATH 649 SYLLABUS

Spring 2009

LECTURER: Tim Carlson

OFFICE: 740 Math Tower

OFFICE HOURS: MW 1:30, F 2:30

TEXT: *A Mathematical Introduction to Logic* by Herbert B. Enderton, second edition.

TOPICS:

We will begin with some basic model theory. What do models of a theory look like? What are their possible cardinalities? What do the definable relations look like?

We then turn to the decidability of theories. In particular, we will show that

- The theory of valid sentences is not decidable.
- The theory of the natural numbers with addition is decidable.
- The theory of the natural numbers with addition and multiplication is not decidable.
- The theory of the universe of sets with the membership relation is not decidable.

Notice that either of the last two results imply that mathematics as a whole is not decidable (since we would assume that either of these two theories is subsumed by mathematics). There is a stronger version of the third result. Matiyasevich, Robinson, Davis and Putnam solved Hilbert's 10th problem by showing

- The set of diophantine equations which have solutions is not decidable.

Another important example of a decidable theory is given by

- The theory of the reals with addition, multiplication and the usual ordering is decidable.

We will touch on these last two results if we have time.

Our final goal will be Gödel's second incompleteness theorem which implies that there is no elementary proof that formal mathematics is without contradictions.

GRADES:

Your grade will be based primarily on the total points you earn in homework assignments given throughout the quarter. Class participation may weigh in for borderline cases.

A final exam is scheduled for Thursday, June 11 at 3:30-5:18 PM. If necessary, we'll use that time to discuss additional material.