SET THEORY AXIOMS

Math 366

1. (Extensionality Axiom) For any sets A and B, if $\forall x(x \in A \text{ iff } x \in B)$ then $A = B$.
2. (Definition of the Emptyset)
   (a) $\emptyset$ is a set.
   (b) For all x, $x \notin \emptyset$.
3. (Definition of Intersection)
   (a) For all sets A and B, $A \cap B$ is a set.
   (b) For all sets A and B, for all x,
   $$x \in A \cap B \text{ iff } x \in A \text{ and } x \in B$$
4. (Definition of Union)
   (a) For all sets A and B, $A \cup B$ is a set.
   (b) For all sets A and B, for all x,
   $$x \in A \cup B \text{ iff } x \in A \text{ or } x \in B$$
5. (Definition of Relative Complement)
   (a) For all sets A and B, $A - B$ is a set.
   (b) For all sets A and B, for all x,
   $$x \in A - B \text{ iff } x \in A \text{ and } x \notin B$$
6. (Definition of $\subseteq$) For all sets A and B,
   $$A \subseteq B \text{ iff } \forall x(x \in A \to x \in B)$$
7. (Definition of Power Set)
   (a) For any set A, $\mathsf{P}(A)$ is a set.

(b) For any set A, for all x,

$$x \in \mathsf{P}(A) \text{ iff } x \subseteq A$$

8. (Pairing Axiom) For all x, y, u and v,

$$(x, y) = (u, v) \text{ iff } x = y \text{ and } u = v$$

9. (Definition of Cartesian Products)

   (a) For all sets A and B, $A \times B$ is a set.

   (b) For all sets A and B, for all x,

$$x \in A \times B \text{ iff } \exists a \in A\, \exists b \in B\, s.t.\ x = (a, b)$$