

MathTime

November 17, 2006

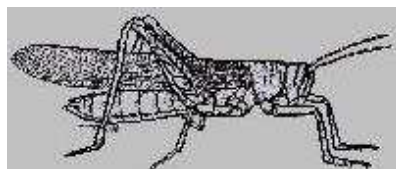
Parity

- Prove that the product of two odd numbers is odd.
 - Prove that the product of an even number with any integer is even.
- Suppose that the sum of two integers is odd. Prove that their product is even.
- Is it possible to get 45045 by multiplying the difference of two integers with their product?
- John with his son and Peter with his son were fishing. John caught the same number of fish as his son, while Peter caught three times as many as his son. Together, they caught 25 fish. How many fish did John catch?
- Is it possible to find two integers a and b such that $7a + 5b = 111$ and $a + b$ is even?
- Ten consecutive natural numbers are written in a row:

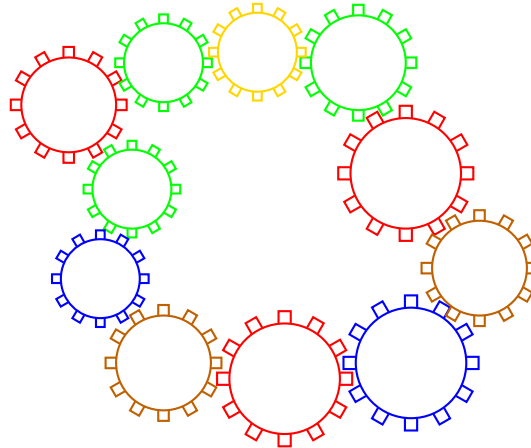
1 2 3 4 5 6 7 8 9 10

Is it possible to place the signs “+” and “−” between them so that the value of the expression will be zero?

- The natural numbers 1, 2, 3, ..., 2005, 2006 are written in a row. Is it possible to place the signs “+” and “−” between them so that the value of the expression will be 2007?
- A grasshopper jumps along a line. His first jump is 1 inch long, his second is 2 inches long, and so on. Each jump can be either to the right or to the left. Can he return to the starting point after 2006 jumps?

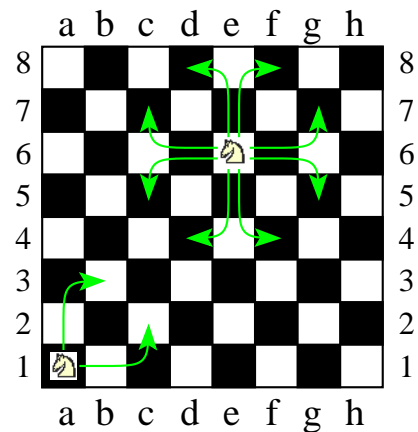


9. Eleven gears are placed on a plane, arranged in a chain as shown. Can all gears rotate simultaneously?



10. On a chessboard, a knight starts from the square $a1$, and returns there after making several moves. Show that the knight makes an even number of moves.

11. Can a knight start at the square $a1$ of a chessboard and go to the square $h8$, visiting each of the remaining squares exactly once on the way?



12. Can an ordinary 8×8 chessboard be covered with 1×2 dominoes so that only the squares $a1$ and $h8$ remain uncovered?