Math 6112 – Spring 2020
Problem Set 5
Due: Friday 14 February 2020

20. Prove that the direct sum of projective modules is projective, i.e. if each P_α is projective, with $\alpha \in I$, then so is $\bigoplus_{\alpha \in I} P_\alpha$.

21. If $e \in R$ is an idempotent (so $e^2 = e$), show that Re is a projective R-module.

22. (Schanuel's Lemma) Suppose we have two short exact sequences

$$0 \to N_1 \to P_1 \to M \to 0$$

and

$$0 \to N_2 \to P_2 \to M \to 0$$

with P_1 and P_2 projective modules. Show that $P_1 \oplus N_2 \simeq P_2 \oplus N_1$.

23. Show that direct summands of injective modules are injective, i.e., if I is an injective R-module and $I = M \oplus N$ then M is injective. (Of course, the same is true for N.)

24. Prove that the direct product of injective modules is again injective, i.e., if each Q_α with $\alpha \in I$ is injective then so is $\prod_{\alpha \in I} Q_\alpha$.