Math 6112 – Spring 2020
Problem Set 8
Due: 20 March 2020

In all problems, let F be a covariant additive functor from $R - \underline{mod}$ to $\mathbb{Z} - \underline{mod}$.

31. Let $(P_\bullet, \varepsilon)$ and $(P'_\bullet, \varepsilon')$ be two projective resolutions of M. Let $\alpha : P_\bullet \to P'_\bullet$ be a lift of the identity morphism $id_M : M \to M$. Show that $\widetilde{F(\alpha_n)}$ is an isomorphism from $H_n(F(P_\bullet))$ to $H_n(F(P'_\bullet))$. Conclude that the left derived functor $L_nF(M)$ is independent of projective resolution *up to isomorphism*.

32. Let $\mu \in Hom_R(M, M')$. In the spirit of the previous problem, determine the dependence of $L_nF(\mu)$ on the choice of projective resolutions of M and M'.

33. (a) Show that L_0F is always right exact.

    (b) Show that if F is right exact then F and L_0F are naturally isomorphic.

34. Let $R = D$ be a commutative PID (such as $\mathbb{Z}$). Let $a \in D$, $a \neq 0$, and let $M = D/(a)$. For any D module N show that

$$Ext^1_D(M, N) \simeq N/aN.$$

    In particular, show that if $N = D/(b)$ then $Ext^1_D(M, N) = D/(a, b)$.

35. In the category of $\mathbb{Z}$-modules, show $Ext^n_{\mathbb{Z}}(-, -) = 0$ for $n > 1$.