Math 580
Supplemental Homework Problems #1

1. Let $X = \mathbb{R}^2$ (the Cartesian plane) and define a relation $(a,b)R(c,d)$ if $a^2+b^2 = c^2+d^2$.
   a. Show that this defines an **equivalence relation**.
   b. Give a geometric description of the **equivalence classes**.
2. Let $X = \{(a,b) \mid a,b \in \mathbb{N} = \{0,1,2,3,\ldots\}\}$ and define a relation $(a,b)R(c,d)$ if $a+d = b+c$.
   a. Show that this defines an **equivalence relation**.
   b. Find a bijection between the set of equivalence classes and $\mathbb{Z}$.
3. Let $X = \{(a,b) \mid a \in \mathbb{Z},\, b \in \mathbb{Z},\, b \neq 0\}$ and define a relation $(a,b)R(c,d)$ if $ad-bc = 0$.
   a. Show that this defines an **equivalence relation**.
   b. Find a bijection between the set of equivalence classes and $\mathbb{Q}$.
4. Which of the following sets and relations define an **equivalence relation**? If it is an equivalence relation, prove it. If not, determine which criterion of an equivalence relation fails to hold.
   a. Let X be the set of all people and define a relation aRb if person a shares a parent with person b.
   b. Let X be the set of all people and define a relation aRb if person a has the same father as person b.
   c. For a non-empty set S, let $X = \mathcal{P}(S)$ and define a relation URV if there exists a injective (one to one) function $f : U \to V$.
   d. For a non-empty set S, let $X = \mathcal{P}(S)$ and define a relation URV if there exists a surjective (onto) function $f : U \to V$.
   e. For a non-empty set S, let $X = \mathcal{P}(S)$ and define a relation URV if there exists a bijection $f : U \to V$.
5. Let X be a set and R be a **relation** that is both symmetric and transitive. Must R be an **equivalence relation**? If R is an equivalence relation, prove it. If not, find an example of a set X with such a relation R that is not an equivalence relation. **Hint:** It might be helpful to think of R as a subset of $X \times X$.