Math 254 Quiz I Solutions Name:___________________________

1. Find the limit, if it exists, or show that the limit does not exist. (10 points).

$$\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2+y^2}$$

Sol) Let $y = ax$.

$$\lim_{\substack{(x,y)\to(0,0)\\ y=ax}} \frac{xy\cos y}{3x^2+y^2} = \lim_{x\to 0} \frac{xax\cos ax}{3x^2+a^2x^2} = \lim_{x\to 0} \frac{x^2a\cos ax}{(3+a^2)x^2} = \frac{a}{3+a^2}$$

i) When $a = 1$, that is, $y = x$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{xy\cos y}{3x^2+y^2} = \frac{1}{3+1^2} = \frac{1}{4}$$

ii) When $a = 2$, that is, $y = 2x$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=2x}} \frac{xy\cos y}{3x^2+y^2} = \frac{2}{3+2^2} = \frac{2}{7}$$

$\therefore$ the limit does not exist.

2. Find the partial derivatives f_x and f_y. (10 points).

$$f(x,y) = \ln\{\sin(e^{(x^2+y)})\} + e^{\ln(\sin(xy))}$$

Sol) By the chain rule

$$f_x(x,y) = \frac{2xe^{(x^2+y)}\cdot\cos(e^{(x^2+y)})}{\sin(e^{(x^2+y)})} + \frac{y\cos(xy)e^{\ln(\sin(xy))}}{\sin(xy)}$$

and

$$f_y(x,y) = \frac{e^{(x^2+y)}\cdot\cos(e^{(x^2+y)})}{\sin(e^{(x^2+y)})} + \frac{x\cos(xy)e^{\ln(\sin(xy))}}{\sin(xy)}$$