

Math 254 Quiz 3 solutions
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Name: _____
Rec. Time: _____
ID# _____

1. Evaluate the following integral. (10 points).

$$\int_0^1 \int_y^1 e^{x^2} dx dy$$

By reversing the order of integration (to do this, you need to look at the region over which you're integrating),

$$\begin{aligned} \int_0^1 \int_y^1 e^{x^2} dx dy &= \int_0^1 \int_0^x e^{x^2} dy dx \\ \therefore \int_0^1 \int_0^x e^{x^2} dy dx &= \int_0^1 ye^{x^2} \Big|_0^x dx = \int_0^1 xe^{x^2} dx = \frac{e^{x^2}}{2} \Big|_0^1 = \frac{e-1}{2} \end{aligned}$$

2. Evaluate the following integral. (10 points).

$$\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} dy dx$$

The region of integration here is the part of the unit circle that lies in the first quadrant. Also, note that $x^2 + y^2 = r^2$. So the given integral becomes

$$\begin{aligned} \int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} dy dx &= \int_0^{\frac{\pi}{2}} \int_0^1 e^{r^2} r dr d\theta \\ &= \int_0^{\frac{\pi}{2}} d\theta \int_0^1 e^{r^2} r dr \end{aligned}$$

$$\begin{aligned} &= \frac{\pi}{2} \int_0^1 e^u du \quad (u = r^2, du = 2r dr) \\ &= \frac{\pi}{2} \frac{(e - 1)}{2} \end{aligned}$$