Math 254 Quiz 4 Solutions Name:__________________________

TA: Moy Easwaran Rec. Time:______________________

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1. $\iint_R \frac{x-2y}{3x-y} dA$, where R is the parallelogram enclosed by lines $x-2y=0,\ x-2y=4,\ 3x-y=1,\ 3x-y=8$. (10 points).

Sol) Let $u = x-2y$ and $v = 3x-y$. Then the line $x-2y=0$ becomes $u=0$, and similarly the other three lines become $u=4$, $v=1$, and $v=8$.

This gives $x = \dfrac{2v-u}{5}$ and $y = \dfrac{v-3u}{5}$, and so $\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} = \dfrac{1}{5}$.

(Alternately, note that $\dfrac{\partial(x,y)}{\partial(u,v)} = \dfrac{1}{\frac{\partial(u,v)}{\partial(x,y)}}$. This gives you the same answer, of course, but the calculations are a lot easier.)

Therefore

$$\iint_R \frac{x-2y}{3x-y} dA = \int_1^8 \int_0^4 \frac{1}{5}\frac{u}{v}\, du\, dv = \frac{1}{5} \cdot \frac{1}{2} u^2 \Big|_0^4 \ln v \Big|_1^8 = \frac{8}{5} \ln 8.$$

2. Find the gradient vector field of f. (10 points).

$f(x,y,z) = e^{\frac{x}{y}} \sin\left(\frac{2}{z}\right)$

Sol) Since $\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$,

then $\nabla f = \frac{1}{y} e^{\frac{x}{y}} \sin\left(\frac{2}{z}\right)\mathbf{i} - \frac{x}{y^2} e^{\frac{x}{y}} \sin\left(\frac{2}{z}\right)\mathbf{j} - \frac{2}{z^2} e^{\frac{x}{y}} \cos\left(\frac{2}{z}\right)\mathbf{k}$