Errata for "Measurability in a Banach space, II" **G. A. Edgar**

Page 563 Line 6. Replace that **by** than

Page 563 Line 11. Delete *3.1*

Page 565 Line 3. Replace 6 **by** 5.6

Page 565. Replace Proposition 3.2 **by**

3.2' Proposition. *If X has the PIP and the Radon-Nikodým property, then X is measure-compact.*

Proof. Let μ be a probability measure on Baire(X, weak). I must show that μ is tight. Let $\varepsilon > 0$. There is $M < \infty$ so that $\Omega = \{ x \in X : \|x\| \leq M \}$ has outer measure $> 1 - \varepsilon$. Define $\mathcal{F} = \{ A \cap \Omega : A \in \text{Baire}(X, \text{weak}) \}$. [This is not, in general, equal to Baire(Ω, weak).] Define a finite measure ν on $\mathcal{F}$ by $\nu(A) = \mu^*(A)$ for all $A \in \mathcal{F}$. Then the function $\varphi \colon \Omega \to X$ defined by $\varphi(x) = x$ is bounded and scalarly measurable. So by the PIP, the vector measure defined by

$$m(A) = \int_A \varphi \, d\nu, \quad A \in \mathcal{F}$$

exists. By the RNP, there is a Bochner integrable function $\psi \colon \Omega \to X$, weakly equivalent to φ, such that

$$m(A) = \int_A \psi \, d\nu, \quad A \in \mathcal{F}.$$

Therefore $\nu = \psi(\nu) = \varphi(\nu)$ is tight, so there is a compact set K with $\mu^*(K) \geq 1 - 2\varepsilon$. This shows that μ is tight. $\square$

Page 572. Replace $(2) \Rightarrow (1)$ **by**

$(2) \Rightarrow (1)$. A completely regular space is realcompact if and only if every zero-one Baire measure is tight. Assume (2) holds, and let μ be a zero-one measure on Baire(X, weak). There is a ball B with positive outer measure, and hence outer measure 1. Let $\Omega = B$, $\mathcal{F} = \{ A \cap B : A \in \text{Baire}(X, \text{weak}) \}$, and define $\widetilde{\mu}$ on $\mathcal{F}$ by $\widetilde{\mu}(A \cap B) = \mu(A)$. Then the identity function $\varphi \colon \Omega \to X$ is a bounded scalarly measurable function, so by assumption it is Pettis integrable. Let $x = \int \varphi \, d\widetilde{\mu}$. We claim that μ is the point-mass ε_x. The measures μ and ε_x agree on all sets of the form $\{ \omega : x^*(\omega) \geq t \}$ which generate Baire(X, weak) [6, Theorem 23], so $\mu = \varepsilon_x$ since both are zero-one measures. This shows that (X, weak) is realcompact. $\square$

Page 573 Line 21. Move *(See note added in proof.)* **to the end of the previous paragraph.**

Page 577 Line -9. Replace e_n **by** e_n^*

Additional Remarks

Theorem 5.9' *For any set Γ, the space $l^1(\Gamma)$ has the Lebesgue-PIP.*

Proof. The proof in the paper (Theorem 5.9) may be used if $\mathfrak{m}_{\mathrm{r}} \geq \mathfrak{c}$ [we only need $\mathfrak{m}_{\mathrm{r}} > \mathfrak{c}$, which is still easier].

If $\mathfrak{m}_{\mathrm{r}} \leq \mathfrak{c}$, then Axiom K holds (Kunen's thesis, Theorem 14.7): There is a set of reals of cardinality $< \mathfrak{m}_{\mathrm{r}}$ which is not Lebesgue measurable and the set of real numbers is not the union of fewer than $\mathfrak{m}_\mathrm{r}$ sets of Lebesgue measure 0. Then, by Fremlin & Talagrand [8, Proposition 3C], *every* Banach space has the Lebesgue-PIP. $\square$

An interesting use of Pettis integral: A. B. Guliasašvili, Estimates for the Pettis integral in interpolation spaces, and a generalization of some imbedding theorems. *Soviet Math. Dokl.* **25** (1982) 428–432.

M. Talagrand, *Pettis Integral and Measure Theory.* Memoirs of the A. M. S. **307**. Among many other results, we have:

16-2-6: $l^1(\Gamma)$ is measure-compact if and only if $\operatorname{card}\Gamma < \mathfrak{m}_\mathrm{r}$. $l^1(\Gamma)$ is realcompact if and only if $\operatorname{card}\Gamma < \mathfrak{m}_2$.

16-4-1 (p. 199): An example X measure-compact, not (C), not Mazur.

G. Vera, Caracterizacion de la Madida Imagen de una Funcion Integrable Pettis, 1985. φ is Pettis integrable if and only if the induced measure λ satisfies:

(*) For every net (Z_j) of convex zero-sets that decreases to the emptyset, $\lambda(Z_j) \to 0$. (In Spanish, so I did not read it.)

R. Pol, Note on the spaces $P(S)$ of regular probability measures whose topology is determined by countable subsets. *Pacific J. Math.* **100** (1982) 185–201. The dual ball of $D[0,1]$ is the continuous image of a first-countable space. (p. 16 and footnote 10)