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Testing Students with Hand-held Technology

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Abstract

Not only has hand-held graphing technology changed what we teach and how we teach, it has also changed how we test our students. For example if your students have access to graphing technology, is there any reason to ask them to graph a function? Graphs of functions containing discontinuities may still be a candidate for testing. These quite often appear to be incorrect on the calculator. Hand-held graphing technology can easily graph quadratic functions. Should you ask your students to graph them? Maybe not, but could they graph the model of the volume of 1 *mm* of water as a function of its temperature by hand? It is

$$V = 0.000008059T^2 - 0.00007999T + 1.00018.$$

Another approach in testing is to ask questions that allow students multiple methods for answering the questions. This usually was not a consideration before the use of hand-held technology permeated mathematics education. Teachers were usually interested in testing a particular algorithm, so the intent was to have students answer the question in only one way. Consider factoring $24x^2 + 71x - 143$ as an example. There are at least two ways of factoring the polynomial. One is the traditional method with pencil and paper, and the other is to use the polynomial zero-factor connection.

Since hand-held graphing technology does many of the traditional skills like graphing, finding zeros (solving equations), factoring, finding maximums/minimums, increasing/decreasing, confirm pencil and paper algorithmic work, etc., test questions may be directed toward concepts behind the above mentioned skills. For example, instead of asking students to graph a quadratic function, ask them to find when it is positive/negative/zero, when it is increasing/decreasing, what is the maximum or minimum, what is the range, etc.

Students Should Have Options for Answering the Question

Below is a sample of a test question that allows students many options for how to answer it – assuming they have been taught mathematics using a function approach with graphing calculators.

Solve the following equation: $x - 3 = \sqrt{2x+2}$.
Several of the technology-based methods are best done by solving the equation above with the equivalent equation $x - 3 - \sqrt{2x+2} = 0$, because students have been taught to look for a value for *x* that causes the function $y_1 = x - 3 - \sqrt{2x + 2}$ to be 0 when they analyzed the behaviors of functions – done before equations are solved.

The **Trace Method** works when the student uses a decimal calculator window and the solution is a "nice" decimal number. These are often found in a beginning algebra course. Shown below in Figure 1, is the solution found using trace and a slightly altered decimal window.

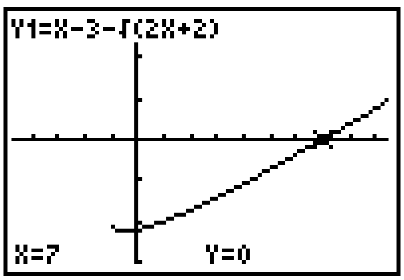

Figure 1

Students know the function will have no more zeros to the right because $x - 3$ is rising faster than $\sqrt{2x + 2}$. Further, there is no need to look to the left of –1 for any zeros because the domain is [–1, ∞).

The **Trace Method** is usually the first method students learn when algebra is taught using a function approach. It flows naturally from the investigation of graphs of functions and is easily understood.

The **Numerical Method** shown below is a numerical refinement of the Zoom-In Method used before calculators had table functionality.
Students see the function $y_1 = x - 3 - \sqrt{2x + 2}$ increasing in value. They must conclude there is a zero for *x* larger than 4. Figure 3 shows the solution to be 7.

X	Y1	
-2	ERROR	
-1	-4	
0	-4.414	
1	-4	
2	-3.449	
3	-2.828	
4	-2.162	

Y1=X-3-√(2X+2)

Figure 2

X	Y1	
4	-2.162	
5	-1.464	
6	-.7417	
7	0	
8	.75736	
9	1.5279	
10	2.3096	

Y1=X-3-√(2X+2)

Figure 3

The **Zeros Method,** using the zero-finder, may be introduced after the algebra instructor is convinced students have mastered simpler methods. Below is the solution, using the zero-finder.

Again, the method depends on student knowing the behavior of functions as described in the Trace Method.

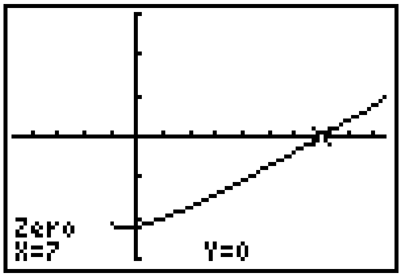

Figure 4

They must know there are no other zeros because of the behavior of the function.
The **Zeros Method** is a little more popular with students because it is often a little faster than other methods – depending on the equation.

Another method that is also often uncovered naturally when mathematical ideas are developed in the context of a problem is the **Intersection Method**. This method requires students to understand that the solution to an equation is a value for *x* that causes the functions on either side of the equal sign to have the same value. Below, in Figure 5, are the graphs of $y_1 = x - 3$ and $y_2 = \sqrt{2x+2}$ showing the use of the intersection finder.

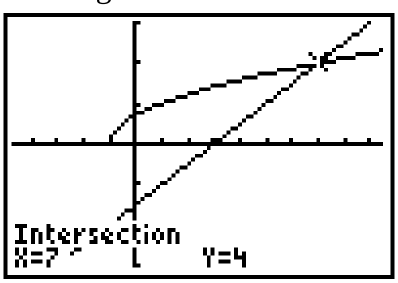

Figure 5

The solution to the equation $x - 3 = \sqrt{2x+2}$ is 7, but some of the poorer students will say the solution is (7, 4), or 4. While this method has strong pedagogical value, students often misuse this method.

The final technology-based method is the **Solver Method**. The problem for the beginning math student is that they must make an educated guess for an answer (cursor location in Figure 6). If it is inappropriate, the solver can yield an incorrect answer. With a guess of 6, the TI-73 solver gives the solution of 7, as shown in Figure 7. This method causes students difficulties when the equation has 2 or more solutions.

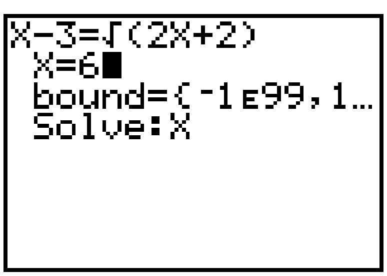

Figure 6

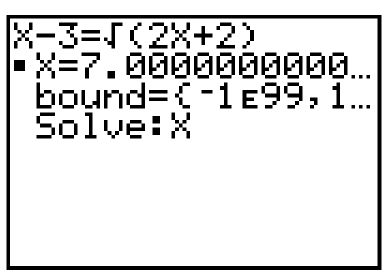

Figure 7

One important strategy in writing a calculator-based test is to use questions that can be answered with a multiplicity of methods. As in the example above, students have at least six methods available.

What to do if no Calculator Use Is Desired for a Particular Content Area

Ask concept or open-ended questions. Students can be inventive and find a way of using a calculator to assist them in answering these questions, however. For example, consider the concept question "If $f(x) = \log x$, and a and b are positive numbers, then $f(a) - f(b)$ is equivalent to

$f(a-b)$, $f(a)f(b)$, $\dfrac{f(a)}{f(b)}$, $f(a+b)$, or $f\left(\dfrac{a}{b}\right)$."

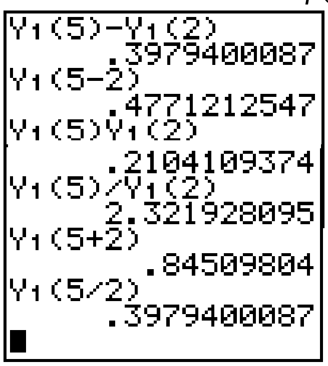

Figure 8

The series of calculations found in Figure 8 is sufficient information for a student to conclude that the fifth response above is correct. Of course, the student luckily chose *a* > *b*. Sometimes concept questions do not cause the desired response from the student – still the correct answer is obtained. Perhaps the question would accomplish the desired results if possible answers were not listed.

Another example of a question designed to exclude the use of a calculator is an open-ended question. Consider the following question "Create a quadratic function that has a maximum value when *x* is 7." There are many correct answers because parameters *d* and *f* in the function $y = d(x+e)^2 + f$ are not specified, other than $d < 0$. This is more difficult for students to use a calculator to assist in answering the question – even a CAS-based calculator.

In some cases, you may want to test the student's ability to properly choose a method – technology or pencil and paper. A simple example is: "Solve the equation $px = 6$." If students choose a technology-based method, it is an inappropriate use of technology. Another type of problem is the case where the calculator yields a nonsense answer. Students must be able to judge the reasonableness of a solution. For example, solve the equation $2^{\frac{1}{x-3}} = 0$.

When the seed 6 is tried in the TI-83 equation solver.

Figure 9

When the seed 2.999 is tried in the TI-83 equation solver.

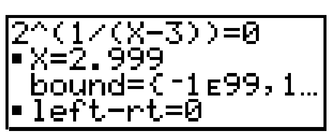

Figure 10

When the seed 2 is tried in the TI-83 equation solver.

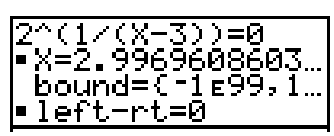

Figure 11

Students will often assume the answer is around 2.999. Some will also conclude the solution is 3. Neither is correct. If they look at a graph of $y = 2^{\frac{1}{x-3}}$, they may still assume an incorrect solution.

Another method for encouraging students to use pencil and paper on a test question is to ask for an exact solution. This idea works when students are using a graphing calculator as opposed to a CAS calculator. Some people split a test into two parts and do not allow students to use the calculator on one part of the test.

To Encourage the use of a Calculator, Ask a Test Question that can't be Answered without Technology.

Assuming an algebra base of knowledge, "Solve the equation $\frac{2x}{x+3} + 2 = \sin x$."
This simple looking equation can be solved quite easily with the use of the graphing calculator as shown in Figure 12.

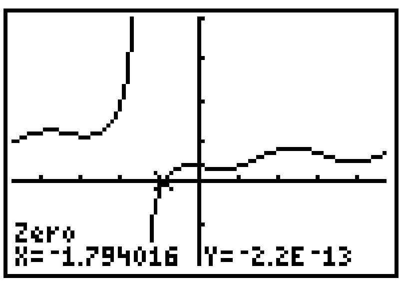

Figure 12

The TI-92 could not find an exact solution, but it did find an approximate solution of –1.794015 using the solve command.

Another example of a test question that encourages a calculator solution is the following: What class of functions can best model the data below: logarithmic, rational, quadratic, exponential, or sinusoidal?

Time as day of the year	1	21	41	61	81	101	121	141	161	…
Minutes of sun light	564	589	621	681	734	786	835	874	897	…

Time as day of the year	181	201	221	241	261	281	301	321	341	361	…
Minutes of sun light	899	877	839	791	740	688	629	595	567	561	…

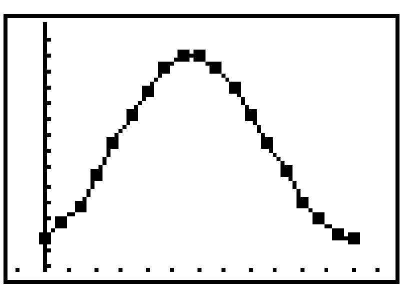

Figure 13

Common sense does not always tell the student that the data is a sinusoid, but using the calculator to graph the data shows that the sinusoidal curve is the best choice.
The problem takes about 1 minute to solve on a calculator. It is unlikely that a pencil and paper solution will take less time.

Graphing Functions by Hand – Still a Test Question

Fifteen years ago, a fair amount of algebra involved the teaching of graphing functions. We taught techniques for graphing each family of functions. But the graphing calculator has diminished the need for such work. It has also allowed us to ask our students to graph more realistic functions like they may find in their applied courses. Below is an example of such a function.

A model for the volume of 1 *mm* of water as a function of its temperature is $V = 0.000008059026887T^2 - 0.0000799990176T + 1.000182722$, graph the model and specify the viewing window that makes the graph "look" like a parabola.

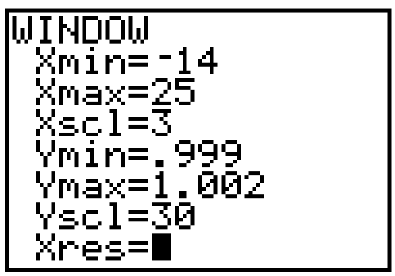

Figure 14

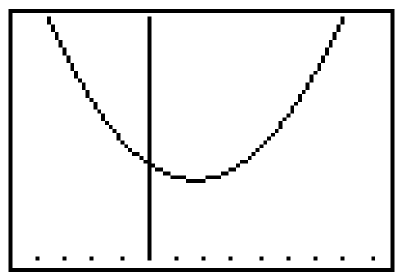

Figure 15

Asking for interpretations of graphs is another way of asking graphing questions. Consider the question below,

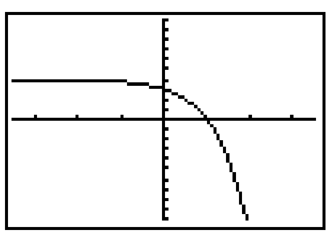

Figure 16

From the graph of $y = f(x)$ in Figure 16, the tick marks on the x-axis are 2 units and on the y-axis they are 1 unit. Estimate $f^{-1}(-4)$.

Not all Assessment is in a Testing Situation

With the use of graphing calculators, a whole new set of instructional material has become available They are commonly called explorations or investigations. These too can be an integral part of the overall assessment of student understanding. Explorations usually require a longer time period and are sometimes best given to students in small groups. Below are a few examples.

Sample 1: Solving complex equations containing rational functions

To be assigned after simpler equations have been solved.

Directions: Solving the next few equations analytically is difficult or impossible, try solving them using a technology-based approach.

1. $\frac{-|x-5|}{x+1} \div \frac{\sqrt{x+2}}{x-1} - \frac{x^2-6x-8}{\sqrt{x^2-4}} = 0$ 1.______________________

2. $\left|\frac{2}{x} - 3\right| = 0$ 2.______________________

3. $\frac{-|x-5|}{x+1} \cdot \frac{\sqrt{x+2}}{x-1} - \frac{x^2-6x+4}{\sqrt{x^2-4}} = 0$ 3.______________________

4. $0 = \left(\frac{2}{x-4} - 3\right)^2$ 4.______________________

5. $\frac{-2}{x-1} + 3 = 2^{x+1} + 7$ 5.______________________

6. $2^{\frac{1}{x-3}} = 0$ 6.______________________

7. $\frac{2^x - 3}{x} = -2$ 7.______________________

8. $\frac{-2}{x-1} + 3 = 2^{x+1} + 7$ 8.______________________

Sample 2: The absolute value transformation

To be assigned after geometric transformations have been taught.

Directions: Describe what the absolute value transformation does to each graph below. In general, describe what the absolute value transformation does to the graph of every quadratic function.

1. Describe the graph of $y = \left|-x^2\right|$ as compared to the graph of $y = -x^2$.

____

2. Describe the graph of $y = \left|-x^2 - 5\right|$ as compared to the graph of $y = -x^2 - 5$.

____

3. Describe the graph of $y = \left|-(x-4)^2\right|$ as compared to the graph of $y = -(x-4)^2$. ____

4. Describe the graph of $y = \left|\frac{1}{2}(x+2)^2 - 3\right|$ as compared to the graph of $y = \frac{1}{2}(x+2)^2 - 3$.

____

Conclusion:____

____

____

Sample 3: Function behavior near zeros

Assigned before the study of polynomial functions.

Directions: Describe the behavior of the graph of each function near the zeros of each function.

1. a. $y = (x-1)^1 (x+3)^2$ ______________________________

1. b. $y = (x-1)^2 (x+3)^1$ ______________________________

1. c. $y = (x-1)^2 (x+3)^2$ ______________________________

1. d. $y = (x-1)^2 (x+3)^3$ ______________________________

1. e. $y = (x-1)^3 (x+3)^4$ ______________________________

2. Find the symbolic representation of any function with a graph that passes through the *x*-axis at –2, 3, and $\frac{3}{5}$, and only touches (is tangent to) the *x*-axis at 5.

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Sample 4: Product and quotient properties of logarithms

To be assigned before the properties are studied.

1. Find log 2 + log 3 and find the logarithm of the product, log (2×3).

__________ __________

2. Find log 4 + log 5 and find the logarithm of the product, log (4×5).

____________ _____________

3. Find log 6 + log 7 and find the logarithm of the product, log (6×7).

____________ _____________

4. Find log 8 + log 9 and find the logarithm of the product, log (8×9).

____________ _____________

5. Find log 42 + log 93 and find the logarithm of the product, log (42×93).

____________ _____________

What conjecture can you make about the logarithm log g + log h?

____

6. Find log 2 – log 3 and find the logarithm of the quotient, log (2/3).

____________ _____________

7. Find log 4 – log 5 and find the logarithm of the quotient, log (4/5).

____________ _____________

8. Find log 6 – log 7 and find the logarithm of the quotient, log (6/7).

____________ _____________

9. Find log 8 – log 9 and find the logarithm of the quotient, log (8/9).

____________ _____________

10. Find log 47 – log 83 and find the logarithm of the quotient, log (47/83).

____________ _____________

What conjecture can you make about the logarithm log g – log h?________

____

Sample 5: Domain of the logarithmic function

To be assigned after properties are studied in class.

Directions: Find the normal domain of the following expressions.

1. $\log(x-4)(x+2)$ and $\log(x-4)+\log(x+2)$ ________ _______

2. $\log(x+7)(x-2)$ and $\log(x+7)+\log(x-2)$ _________ _______

3. $\log\dfrac{(x-2)}{(x+3)}$ and $\log(x-2)-\log(x+3)$ ______________ _______

4. $\log\dfrac{(x+5)}{(x-1)}$ and $\log(x+5)-\log(x-1)$ _______________ _______

5. Make a conjecture about the use of the product and quotient properties of logarithms.

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Conclusion: There are many strategies for testing students using hand-held calculator technology. Above are a few samples. One can conclude that we must rethink how to test in light of using technology.