National Council of Teachers of Mathematics 82nd Annual Meeting

The Meaning of "A Function Approach to Teaching Algebra"

Thursday, April 22, 2004
8:00 am - 9:00 am
Session 26, Franklin Hall B1
Philadelphia Marriott

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Description of a Function Approach:

Teaching function notation early is not teaching from a function approach!

We will start with numeric representations of functions and teach students to freely move through representations and then analyze function behaviors with parameter-behavior connections.

Once accomplished, we can then "use" function to teach mathematics such as factoring, equation solving, systems of equations, inequalities, properties of inequalities, definitions, etc. This is teaching from a function approach!

Integration of the Function Approach into a Traditional Curriculum*

- Start with data pairs from the real world (no traditional symbol manipulation needed) [numerical & attention]

- Teach students to move freely from numeric to graphic and make the connections between the two [numerical & visual]

- Teach students to move from numeric or literal to symbolic – see Implementation Document at http://www.math.ohio-state.edu/~elaughba/amatyc/2002/2002_algebra_using_function_approach_implementation.pdf [numerical, visual, & pattern]

- Teach the connection of all these to real world situations (through contextual data) [associations & attention]

- Teach the behaviors of basic functions (increasing/decreasing, max/min, rate of change, zeros, when positive/negative, domain, and range [associations & enriched environment]

- Teach parameter-behavior connections [numerical, visual, associations, & enriched environment]

(All of above takes 6-8 days)

- When finished, you can **"use" function, function representation, and function behaviors** in the teaching of more traditional topics. [numerical, visual, associations, & enriched environment]

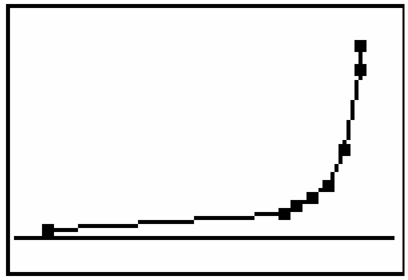
Table of Contents for the Function Implementation Module

Numeric and Graphical Representation Identification of General Shapes

EXAMPLES:

L1	L2	L3 3
1000	.2	
1750	.8	
1800	1	
1850	1.2	
1900	1.7	
1950	2.8	
1994	5.4	

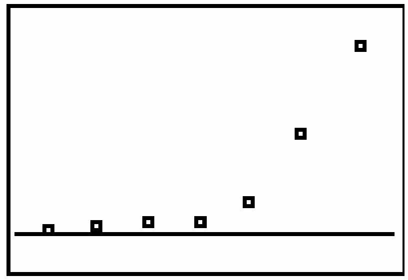
L3(1)=



Calendar years vs. world population in billions
It looks like a “J” and appears to be increasing

L1	L2	L3 3
1940	51	
1950	257	
1960	291	
1970	381	
1980	909	
1990	3113	
2002	6000	

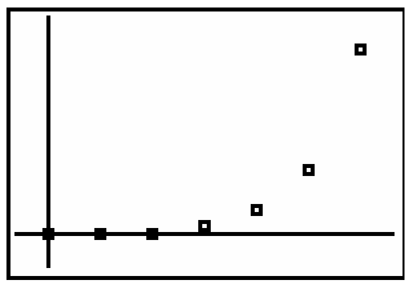
L3(1)=



Calendar years vs. US debt
It looks like a “J” and appears to be increasing

L1	L2	L3 3
0	5000	
30	14142	
60	40000	
90	113137	
120	320000	
150	905097	
180	2.56E6	

L3(1)=



Time in minutes vs. number of E. coli in a Petri dish
It looks like a “J” and appears to be increasing

L1	L2	L3 3
1940	3.3	
1950	8.9	
1960	23.9	
1970	68.5	
1980	165.6	
1990	377.5	
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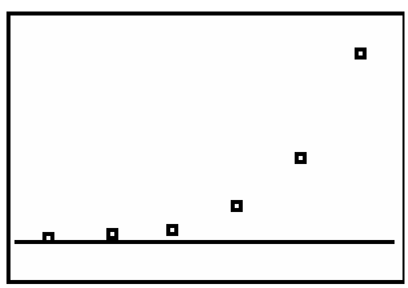
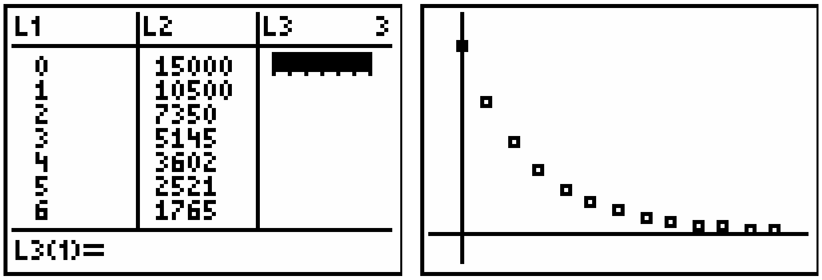
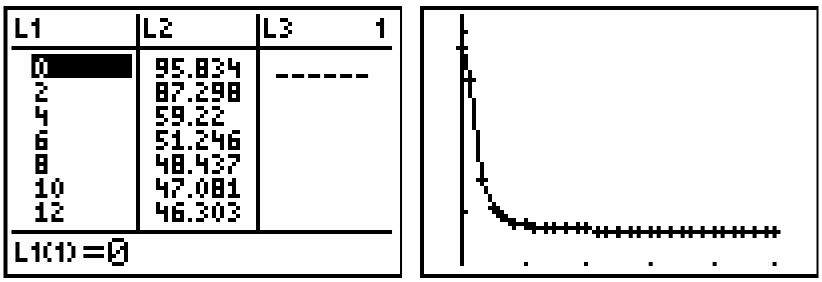
L3(1)=



Time in calendar years vs. US education expenses in billions
It looks like a “J” and appears to be increasing



Time in years vs. value of a car.
It looks like a backward “J” and appears to be decreasing



Time in seconds vs. temperature as a temperature probe cools
It looks like a backward “J” and appears to be decreasing

Keep in mind that in class, the activity would have several function types with the same question, “What shape is the data relationship and does it appear to be increasing or decreasing?” At this point we have gotten the attention of most students because we used technology and a real world context. Students know that real world relationships can have a definite pattern and seem to increase or decrease (or sometimes both). They have seen two representations of functions.

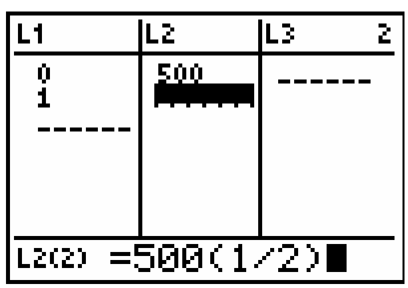
Symbolic Representation Connection:

Linear
Quadratic
Exponential
Rational

EXAMPLE:

Suppose we have 500 M & M candies (the initial condition) and we toss them on the table. How many do we expect to have the M facing up?

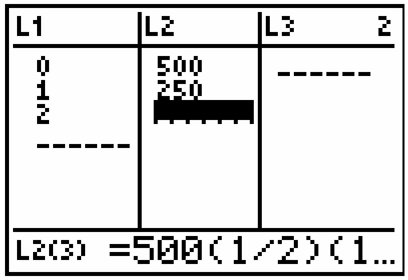
Students say: about 250.You say, "How did you get that?" (see edit line for student answer)



We eat the M & M's with the M facing up, and toss the remaining on the table. We now have about 250 M & M candies on the table. How many do we expect to have the M facing up?

Students say: 125 — You say, "How did you get that?"

Students say: 250(1/2) — You say, "But where did the 250 come from?" (see edit line for student answer)



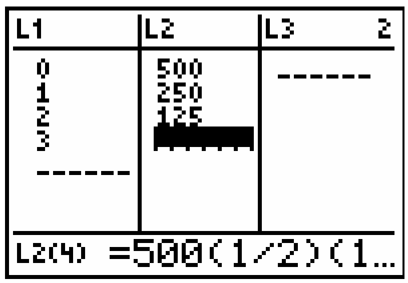
Of the M & M's with the M facing up – we eat them and toss the remaining on the table. We now have about 125 M & M candies on the table. How many do we expect to have the M facing up?

Students say: about 63 You say, "How did you get that?"

Students say: 125(1/2) You say, "But where did the 125 come from?"

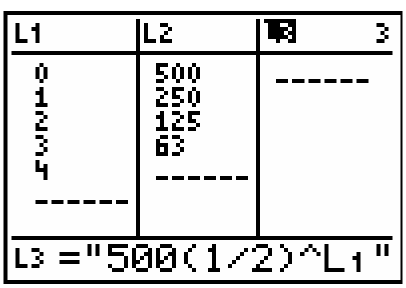
Students say: 250(1/2)(1/2) You say, "But where did the 250 come from?"

See the edit line for the student answer.

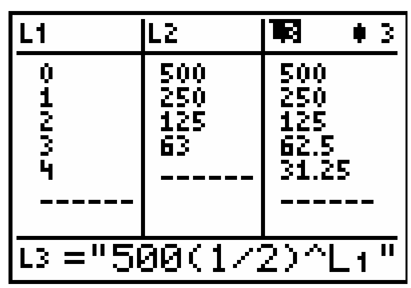


(Note: There are now 3 factors of ½)

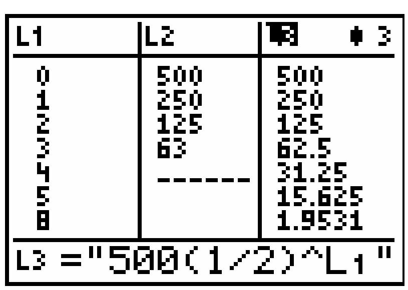
At this point most of the class recognizes the pattern and you are ready for the introduction of symbols – see below.



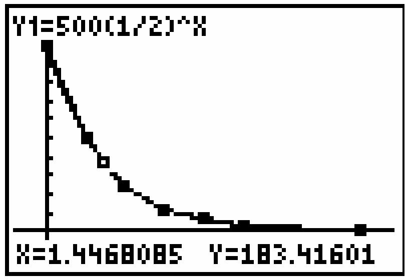
and then:



In L1, you can enter new values to see the power of abstract mathematical symbols.



We now need to look at the graphical representation and use the Y= editor to introduce standard x notation.



This process can be used for a variety of functions such as linear, quadratic, and rational.

Geometric Behaviors of Functions

Increasing/Decreasing
Maximum/Minimum
Zeros
Positive/Negative
Domain/Range
Average Rate of Change

EXAMPLE: See http://www.math.ohio-state.edu/~elaughba/ then look in the group file (Foundations.8xg) for ZEROSPN.8xv or Zeros, Pos & Neg on the graphing calculator.

Parameter-Behavior Connections

EXAMPLE: See http://education.ti.com/educationportal/activityexchange/activity_detail.do?cid=us&activityid=1551

The Payoff

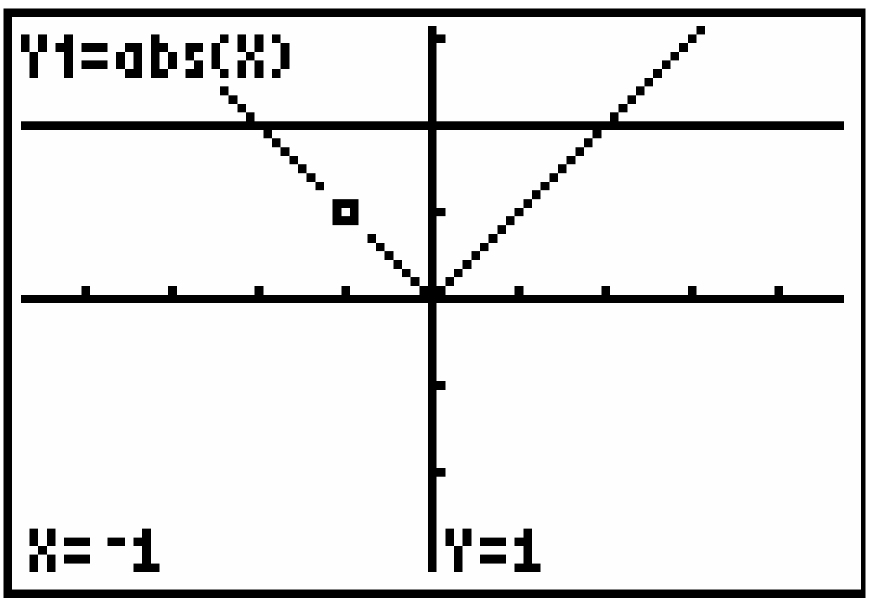
- Adding and Subtracting Polynomial Functions
- Multiplying Polynomial Functions
- Laws of Exponents
- Reducing Rational Expressions
- Function Notation with Technology
- Factoring, see http://education.ti.com/educationportal/activityexchange/activity_detail.do?cid=us&activityid=1565
- Modeling
- Solving Equations
  Trace Method
  Numeric Method
  Zeros method
  Intersection Method
- Absolute Value Inequality Property – See next page for example:
- Asymptotic Behavior
- Laws of Exponents
- Etc.

FIRST? (no)

If $|x| \leq a$ for some positive number a

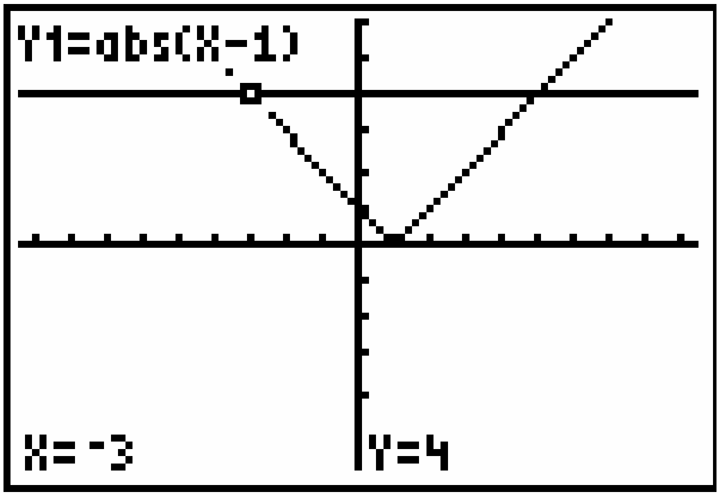
Then $-a \leq x \leq a$

OR THIS FIRST? (absolutely)



Now use trace to figure out why the property makes sense.

But how do I solve $|x-1| \le 4$ using algebra?



Now use trace to figure out the solution is

$$-4 + 1 \le x - 1 + 1 \le 4 + 1$$

Five Reasons for Using a Function Approach:

Disclaimer: Just like educational research is open to interpretation, so is neuroscientific research.

Improved memory through Associations:

In the function approach, there are so many more ways of making associations (which help memory) than in the equation-solving approach. Nearly every topic is "associated" with something else in function and through function, and more importantly, these associations are actually used in the teaching process, and are not “add-on” memory techniques to the teaching process.

Schacter, D. L. (2001). *The seven sins of memory: How the mind forgets and remembers*, 31-33. Houghton Mifflin Company. Boston.

Wagenaar found that the more cues he provided, the more likely he was to remember key details of the event.

Schacter, D. L. (2001). *The seven sins of memory: How the mind forgets and remembers*, 63. Houghton Mifflin Company. Boston.

[my description]

Schacter (p.63) describes an experiment used in the field of psychology called the Baker/baker experiment. Remembering a proper name, like Baker, is extremely difficult because "Baker" tells you nothing at all about the person Baker and is easily blocked from recall. This is much like teaching the First Law of Exponents (or any other algebraic process) when you do not associate it with anything and simply treat it as a mathematical process. The name Baker, or the First Law of Exponents (taught as a process in isolation) do not imply any characteristics of association for remembering them in the future – let alone being able to use the Law. BUT, the name baker (a person who bakes) is rich with associated cues and is much easier to remember and is less affected by transience or blocking memory loss.

This concept is a major pedagogical tool when teaching algebra using a function approach. In teaching equation solving from a function approach (as one example), you use function-based methods first because this associates equation-solving (the new stuff to be learned) with what students have already learned about the zero behavior of a function, about the relationship to the *x*-axis (using the zeros method), about the numeric representation table on the graphing calculator (using the numeric method), about the intersection of lines (using the intersection method), about how each side of the equation is a function (something they are familiar with), about how the context makes the solution have a real-world meaning (with its own rich associative cues), about the consistency of using the graphing calculator to do mathematics, and then yes finally, the symbol manipulations become one more cue as to the meaning and process of solving an equation.

Schacter, D. L. (2001). *The seven sins of memory: How the mind forgets and remembers*, 34. Houghton Mifflin Company. Boston.

Any attempt to reduce transience should try to seize control of what happens in the early moments of memory formation, when encoding processes powerfully influence the fate of the new memory.

Byrnes, J. P., (2001). *Minds, brains and learning: Understanding the psychological and educational relevance of neuroscientific research*, 53. The Guilford Press. New York.

Studies suggest that when a given record [memory/learning] is activated, the activation of this record seems to "spread" to other records associated with it.

Schwartz, J. M. & Begley, S. (2003) *The mind and the brain: Neuroplasticity and the power of mental force*, 326-327. ReganBooks/HarperCollins Publishers. New York.

If the words you hear or the images you see are associated with a poignant memory, for instance, then they trigger – automatically and without any active effort by you – more attention than stimuli that lack such association.

[more later on attention]*

Minds are Visual

The mind favors learning through a visual approach. So this increases the odds that a person can learn algebra if taught through the function approach. This fact does perhaps imply that one reason students do not learn algebra well through the equation-solving approach is that we are not hard-wired to learn through abstract symbols.

Pinker, S. (1997). *How the mind works*, 359-360. W. W. Norton & Company. New York.

Thanks to graphs, we primates grasp mathematics with our eyes and our mind's eye. Functions are shapes (linear, flat, steep, crossing, smooth), and operating is doodling in mental imagery (rotating, extrapolating, filling, tracing). So, vision was co-opted for mathematical thinking, …

Pinker, S. (1997). *How the mind works*, 298. W. W. Norton & Company. New York.

At some point between gazing and thinking, images must give way to ideas.

Pinker, S. (1997). *How the mind works*, 191. W. W. Norton & Company. New York.

Even scientists, when they try to grasp abstract mathematical relationships, plot them in graphs that show them as two- and three-dimensional shapes. Our capacity for abstract thought has co-opted the coordinate system and inventory of objects made available by a well-developed visual system.

Schacter, D. L. (2001). *The seven sins of memory: How the mind forgets and remembers*, 103. Houghton Mifflin Company. Boston.

… after studying pictures along with the words, participants expect more from their memories. They easily reject items that do not contain the distinctive pictorial information they are seeking …

Scott, T. (2000). *How to think like Einstein*, 200, Barns & Noble. New York.

Our minds have a remarkable ability to remember images. Even if you can't remember your brother's phone number, you can store enough images to choke a computer.

We Also Use a Number Sense

Since the mind also has an innate number sense, we capitalize on this by using the numeric representations through-out teaching a function approach, connecting numbers (parameters) to behaviors, and using number to develop symbolic form.

Pinker, S. (1997). *How the mind works*, 340-341. W. W. Norton & Company. New York.

David Geary has suggested that natural selection gave children some basic mathematical abilities: determining the quantity of small sets, understanding relations like "more than" and "less than" and the ordering of small numbers, adding and subtracting small sets, and using number words for simple counting, measurement, and arithmetic.

How can people use their Stone Age minds to wield high-tech mathematical instruments? The first way is to set mental modules to work on objects other than the ones they were designed for. Ordinarily, lines and shapes are analyzed by imagery and other components of our spatial sense, and heaps of things are analyzed by our number faculty.

We Use an Enriched Environment

In teaching algebra through a function approach using graphing calculators, we can provide a much more enriched teaching environment than through using an equation-solving approach. One reason is that graphing technology is used as a tool of exploration in a function approach. An enriched teaching environment means having diverse techniques available for learning. The enriched environment causes more learning and less forgetting than does a traditional equation-solving approach with its "one way" of doing algebra.

Marcus, G. (2004). *The birth of the mind*, 98-99. Basic Books. New York.

In living organisms, the brains of rats and mice that are raised in complex, toy-filled environments have, in comparison to rodents raised in ordinary, drab cages, thicker cortical tissue, more intricately branched dendrites, and more synapses per neuron. Learning, whether in a rat or a human, is a process by which experience modifies the brain by modifying the expression of genes.

Restak, R. M. (2003). *The new brain: How the modern age is rewiring your mind*, 32. Rodale.

… intelligence is plastic and modifiable. All of our experiences result in the formation of neuronal circuits. The richer, more varied, and more challenging the experiences, the more elaborate the neuronal circuits. … the brain's capacity for change-its plasticity-remains vigorous throughout our lives.

Paying Attention

Typically, the use of technology and a context will focus attention on the task at hand – provided the task is directly relevant to the algebra being learned.

Schwartz, J. M. & Begley, S. (2003) *The mind and the brain: Neuroplasticity and the power of mental force*, 224. ReganBooks/HarperCollins Publishers. New York.

And therein lies the key. Physical changes [learning] in the brain depend for their creation on a mental state in the mind – the state called attention. Paying attention matters. It matters for the dynamic structure of the very circuits of the brain and for the brain's ability to remake itself.

Byrnes, J. P., (2001). *Minds, brains and learning: Understanding the psychological and educational relevance of neuroscientific research*, 55. The Guilford Press. New York.

Attention is crucial to memory because information cannot enter the memory system unless it is "attended to."

Schwartz, J. M. & Begley, S. (2003) *The mind and the brain: Neuroplasticity and the power of mental force*, 285. ReganBooks/HarperCollins Publishers. New York.

An examination of the mathematics, Stapp argued, shows that "the conscious intentions of a human being can influence the activities of his brain…"

Schwartz, J. M. & Begley, S. (2003) *The mind and the brain: Neuroplasticity and the power of mental force*, 329. ReganBooks/HarperCollins Publishers. New York.

An activity usually deemed to be a property of the mind – paying attention – determines the activity of the brain. Attention can do more than enhance the responses of selected neurons. It can also turn down the volume in competing regions. Ordinarily – that is, in the absence of attention – distractions suppress the processing of a target stimulus.

Byrnes, J. P., (2001). *Minds, brains and learning: Understanding the psychological and educational relevance of neuroscientific research*, 74. The Guilford Press. New York.

Apart from being intensively studied by psychologists and neuroscientists for quite some time, attention can also be considered a "gateway to learning." All the major theorists in the area of learning agree that information in a lesson cannot be learned if children are no paying attention.

Two Possible Reasons not to use the Symbolic Approach:

Teaching symbol manipulation does NOT translate to understanding algebra.

Pinker, S. (1997). *How the mind works*, 93. W. W. Norton & Company. New York.

[in reference to the thought experiment called the Chinese Room] Therefore, understanding-and by extension, any aspect of intelligence-is not the same as symbol manipulation or computation.

When using the function approach, there are NO (or minimal, depending on teacher behavior) negative mental conflicts with what and how students learned algebra the first time. This enhances learning, and more importantly, does not block learning like when students are re-taught algebra using the equation-solving approach.

Schacter, D. L. (2001). *The seven sins of memory: How the mind forgets and remembers*, 33. Houghton Mifflin Company. Boston.

Experiences that are similar to those we wish to remember create interference that impairs memory.

Byrnes, J. P., (2001). *Minds, brains and learning: Understanding the psychological and educational relevance of neuroscientific research*, 59. The Guilford Press. New York.

Sometimes an *interference* relation can develop between information already in memory and information that we are just learning.