TRANSFER OF ORBITAL INTEGRALS AND DIVISION ALGEBRAS

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Let F_u be a local non-archimedean field, G_u the multiplicative group of a division algebra D_u central of rank n over F_u, and $G'_u = GL(n, F_u)$. There is an embedding of the set of conjugacy classes γ in G_u as the set of elliptic conjugacy classes γ' in G'_u, defined by $p_\gamma = p_{\gamma'}$; here p_γ is the characteristic polynomial of γ, and $p_{\gamma'}$ is that of γ'. In a fundamental but unpublished work [DK] (see also [BDKV]) of the late 1970's, Deligne and Kazhdan proved:

THEOREM. *There is a bijection from the set of equivalence classes of irreducible G_u-modules π_u to the set of equivalence classes of irreducible square-integrable G'_u-modules π'_u, defined by the character relation $\chi_{\pi'_u}(\gamma') = (-1)^{n-1}\chi_{\pi_u}(\gamma)$ for every regular γ in G_u with image γ' in G'_u. Here $\chi_{\pi'_u}$ denotes the character [H] of π'_u, and χ_{π_u} that of π_u.*

By virtue of [K2], it suffices to prove this for F_u of characteristic zero. In fact, all of our arguments hold also in the positive characteristic case, except for the reference [K1] to the orthonormality relations for characters used in the proof of Proposition 5. These relations are known to follow once the local integrability of the characters is established also in the positive characteristic case (1996 update: this has now been done in [L]). The Theorem had been proven in [JL] for $n = 2$. The proof of [JL], as well as that of [DK], relies on global techniques, principally the Selberg trace formula, and on local studies of transfer of orbital integrals between G_u and G'_u. There are several proofs of this local transfer; see the exposition [R] to [DK], where germs and buildings are used, or [F1], where the relations between germs of characters and orbital integrals (due to [Ho], [H], [K1]) are exploited. The purpose of the present note is to prove the Theorem without transferring locally the orbital integrals (except in a trivial case), and *consequently deduce this transfer* (see the Corollary below) *by global means*. These means include, in addition to the trace formula, the Hecke L-function theory of [GJ]. The observation that the transfer of orbital integrals can be deduced from the lifting Theorem was already made in the context of [FK1], §27.3. The deduction relies on results of [BDK] and [K1].

The point in our present proof is that the theory of L-functions is used to show the finiteness of the set of representations which appear in the trace formulae, under some conditions. This observation was made already in [DK] (see [BDKV], pp. 78-82), which was concerned not only with the Theorem, but also it contained a discussion – in the context of $GL(n)$ – of some fundamental ideas later developed in [BDK] and [K1]. Here we show that this observation suffices to complete the proof of the Theorem and Corollary. We then obtain a simple proof of the Deligne-Kazhdan theorem in the division algebra case. This Theorem is used as the first step in the inductive proof of the theorem in the simple

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algebra case (see [BDKV], as completed in [F1]). The wish to provide such a simple proof (essentially due to [DK]) to this first induction step used in [F1; III], and consequently to dissipate some misconceptions concerning the difficulty of this case, were a main motivation for us to write this note. Namely, all of our arguments can be found in [DK] or [R], and in particular the usage of the Hecke theory, but it was assumed in [R] that the transfer of orbital integrals had to be proven first, and this led to a proof longer and more complicated than necessary.

As noted by the referee, our proof is not elementary. It uses deep theorems in harmonic analysis (e.g. Kazhdan's orthonormality relations for characters). Consequently the reader should have a basic knowledge of representation theory to understand the proof. However our reduction of the correspondence to "standard theorems" is considerably shorter than the other reductions. At present the L-function theory is sufficiently developed for our purposes only in the case of $GL(n)$ and the multiplicative group of a division algebra. In particular it applies also in the analogous situation of base-change for $GL(n)$ (proven by Arthur-Clozel), where the local theory of base-change lifting for $GL(n)$ can be established using purely global means such as the trace formula and the L-functions theory of [JS]; transfer of orbital integrals is obtained as a corollary (see [F2]). It will be interesting to develop this Hecke theory for other groups, for example to satisfy the needs of the metaplectic correspondence, symmetric-square lifting from $SL(2)$ to $PGL(3)$, or base-change from $U(3,E/F)$ to $GL(3,E)$.

In the global proof one takes a number field F, totally imaginary for simplicity, which has a place u such that the completion of F at u is the local field F_u in the Theorem. Fix a finite place $u' \neq u$ of F. Let D be a division algebra central of rank n over F, whose invariant $inv_u D$ at u is equal to the invariant $inv D_u$ of D_u (equivalently $D_u \simeq D \otimes_F F_u$), and such that $inv_v D = 0$ for all $v \neq u, u'$. Then $D \otimes_F F_v$ is the matrix algebra $M(n, F_v)$ for every $v \neq u, u'$. Put $G_v = (D \otimes_F F_v)^\times$ and $G'_v = GL(n, F_v)$ for every place v of F. Note that the multiplicative group G of D is an inner form of $G' = GL(n)$. Choose an F-rational invariant differential form of maximal degree on G; it defines Haar measures dg_v on G_v and dg'_v on G'_v for all v, and product measures $dg = \otimes dg_v$ on $G(\mathbb{A})$ and $dg' = \otimes dg'_v$ on $G'(\mathbb{A})$. Note that the center Z of G is isomorphic to that of G', and to the multiplicative group. To simplify the notations we deal here only with representations and functions which transform trivially under the center.

The trace formula is stated for a function $f = \otimes f_v$ in $C_c^\infty(\overline{G}(\mathbb{A}))$. We put $\overline{G}$ for G/Z. It involves orbital integrals

$$\Phi(\gamma, f) = \int_{\overline{G}(\mathbb{A})/G_\gamma(F)} f(g\gamma g^{-1})dg = |G_\gamma(\mathbb{A})/Z(\mathbb{A})G_\gamma(F)| \int_{G(\mathbb{A})/G_\gamma(\mathbb{A})} f(g\gamma g^{-1})dg,$$

and traces

$$tr\ \pi(f) = \prod_v tr\ \pi_v(f_v), \text{ where } \pi_v(f_v) = \int_{\overline{G}_v} f_v(g)\pi_v(g)dg.$$

We take the component f_u to be supported on the set of γ in G_u such that γ^n is regular. Then for γ in $G(F)$ we have $\Phi(\gamma, f) \neq 0$ only when γ is regular, in which case the centralizer G_γ of γ in G is a torus.

TRACE FORMULA FOR G**.** *For any* f *as above we have*

$$\sum_{\gamma} \Phi(\gamma, f) = \sum_{\pi} \; m(\pi) \; tr \; \pi(f). \tag{1}$$

The sum on the left ranges over the set of conjugacy classes γ *in* $\overline{G}(F)$ *such that* γ^n *is regular. The sum on the right ranges over the set of equivalence classes of automorphic* $G(\mathbb{A})$*-modules* π *with trivial central character;* $m(\pi)$ *denotes the multiplicity of* π *in the space of automorphic forms.*

The proof of this is elementary (cf. [F1], §I.3, pp. 143/4).

The trace formula for $G'(\mathbb{A})$ will be stated for a function $f' = \otimes f'_v$ in $C_c^\infty(\overline{G}'(\mathbb{A}))$, with the following properties. Fix a finite place $u'' \neq u, u'$ of F, and let $f'_{u''}$ be a normalized coefficient of a supercuspidal $G'_{u''}-$module $\pi^0_{u''}$. Thus $tr\pi_{u''}(f'_{u''}) = 0$ for any irreducible $\pi_{u''}$ inequivalent to $\pi^0_{u''}$, and $tr\ \pi^0_{u''}(f'_{u''}) = 1$. Let $f'_{u'}$ be a pseudo-coefficient (see [K1]) of the Steinberg $G'_{u'}$-module $st_{u'}$. Then $trst_{u'}(f'_{u'}) = 1$, and $tr\ \pi_{u'}(f'_{u'}) = 0$ for every irreducible tempered $G'_{u'}$-module $\pi_{u'}$ inequivalent to $st_{u'}$. Moreover, the orbital integrals of $f'_{u'}$ vanish on the regular non-elliptic set, and

$$'\Phi(\gamma, f'_{u'}) = \int_{\overline{G}'_{u'}} f'_{u'}(g\gamma g^{-1})dg$$

is equal to $\chi_{st_{u'}}(\gamma) = (-1)^{n-1}$ on the regular elliptic set. Finally let f'_u be a function supported on the set of γ in G'_u such that γ^n is regular. By [F1], §I.3, Cor. (the discussion of [F1], pp. 143/4, leading to this Corollary, is self-contained and simple, especially on noting that the Prop. on p. 143 of [F1] is needed only in the case where G of [F1] is an inner form of $GL(n)$; there is no need to reproduce here this discussion), we have

TRACE FORMULA FOR G'**.** *For any* f' *as above we have*

$$\sum_{\gamma'} \Phi(\gamma', f') = \sum_{\pi'} tr\ \pi'(f'). \tag{2}$$

The sum on the left ranges over the set of elliptic regular conjugacy classes γ' *in* $\overline{G}'(F)$ *such that* γ^n *is regular. On the right the sum ranges over the set of cuspidal* $G'(\mathbb{A})$*- modules* π' *with trivial central character.*

Note that the multiplicity of each such π' in the cuspidal spectrum for G' is one.

The trace formula for G' will be used with a function $f' = \otimes f'_v$ whose components at u' and u'' are as described above. The component f'_u is taken to be supported on the set of γ in G_u with regular γ^n; moreover we assume that its orbital integrals vanish on the non-elliptic set of G'_u. In this note we call such f'_u a *regular-discrete* function. The isomorphism $G_v \simeq G'_v$ for $v \neq u, u'$, can be and is used to transfer f'_v to a function f_v on G_v. Let $f_{u'}$ be a normalized matrix coefficient of the trivial $G_{u'}$-module $\mathbf{1}_{u'}$. Then $tr\ \mathbf{1}_{u'}(f_{u'}) = 1$, and $tr\ \pi_{u'}(f_{u'}) = 0$ for any irreducible $\pi_{u'}$ inequivalent to $\mathbf{1}_{u'}$. Moreover $'\Phi(\gamma, f_{u'}) = 1$ for all γ in $G_{u'}$. Finally, take f_u to be a regular-discrete function on G_u (namely f_u is supported on the set of γ in G_u such that γ^n is regular), with $'\Phi(\gamma, f_u) = \ '\Phi(\gamma', f'_u)$ for every γ regular in $G_u; \gamma'$ is the image of γ in G'_u. We say in this case that f_u and f'_u have *matching orbital integrals*, and note that it is a well-known, simple result of Harish-Chandra that for every regular-discrete f'_u on G'_u there exists such f_u on G_u, and for every regular-discrete f_u on G_u there exists such f'_u on G'_u, with matching orbital integrals. The existence of matching functions in general is a more difficult problem, which we solve below on using the Theorem; its solution is not required for the proof of the Theorem.

PROPOSITION 1. *For the $f' = \otimes f'_v$ and $f = \otimes f_v$ related as above, we have*

$$\sum_{\pi'} tr\ \pi'(f') = \sum_{\pi} m(\pi)\ tr\ \pi(f). \tag{3}$$

The sums are those of (**1**) *and* (**2**).

Proof. By the choice of f and f', the sums over γ and γ' in (**1**) and (**2**) range over isomorphic sets ($\gamma \leftrightarrow \gamma'$ iff $p_\gamma = p_{\gamma'}$), and $\Phi(\gamma, f) = \Phi(\gamma', f')$ for all $\gamma \leftrightarrow \gamma'$. Note that for regular γ the centralizers G_γ of γ in G, and $G'_{\gamma'}$ of γ' in G', are isomorphic elliptic tori; this isomorphism is used to transfer measures between these groups. The proposition follows.

Let π'_{0u} be a square-integrable G'_u-module. By a standard result, see [F1], Prop. III. 3, p. 173, there exists a cuspidal $G'(\mathbb{A})$-module π'_0 whose component at u is the chosen π'_{0u}, at u' it is the Steinberg $st_{u'}$, and at u'' it is the supercuspidal $\pi^0_{u''}$. Denote by $\mathbb{A}^u$ the ring of F-adeles without u-component. Denote by $\pi^u_0 = \underset{v\neq u}{\otimes} \pi^u_{0v}$ the $G(\mathbb{A}^u)$-module $\mathbf{1}_{u'} \otimes (\underset{v\neq u,u'}{\otimes} \pi_{0v})$. Here we identify π'_{0v} with a G_v-module π_{0v} for $v \neq u, u'$, by $G_v \simeq G'_v$. The standard-type isolation argument of [F1], Prop III. 4, pp. 174/6, implies

PROPOSITION 2. *For the given square-integrable π'_{0u} there exist irreducible G_u-modules π_u, such that for any matching regular-discrete f_u and f'_u, we have*

$$(-1)^{n-1} tr\ \pi'_{0u}(f'_u) = \sum_{\pi_u} m(\pi_u \otimes \pi^u_0)\ tr\ \pi_u(f_u). \tag{4}$$

REMARK. In the proof of (**4**) it is worthwhile to note that the choice of $f'_{u''}$ in (**3**) implies that the π' of (**3**) are all cuspidal. Hence each component of π' is non-degenerate; by [Z], (9.7b), if $tr\ \pi'_{u'}(f'_{u'}) \neq 0$ then $\pi'_{u'} \simeq st_{u'}$ and so $tr st_{u'}(f'_{u'}) = (-1)^{n-1}$.

The usual argument, of [JL], [DK], [R], [F1], to deduce the Theorem from (**4**), is based on evaluation of (**4**) at f_u which is a normalized coefficient of some π_u which occurs in (**4**) with $m(\pi_u \otimes \pi^u) \neq 0$. To do this, one has to show that there exist f'_u with orbital integrals matching those of f_u. We shall argue differently. Using the Hecke theory of [GJ] we prove (following [DK]) that the sum in (**4**) is finite *uniformly* in f_u. In fact, since f_u is biinvariant under some compact open subgroup K_u of the compact (modulo Z_u) group G_u, there are only finitely many π_u with $tr\pi_u(f_u) \neq 0$. However the size of the finite set of such π_u increases as K_u decreases, and a-priori the sum in (**4**) may be infinite (for a variable f_u). In order to use the orthonormality relations for characters (see the passage from Prop. 4 to Prop. 5 below) we need to know that the sum in (**4**) (and so in (**6**) below) is finite uniformly in, or independently of, f_u. Thus we prove

PROPOSITION 3. *The sum over π_u in* (**4**) *is finite (uniformly in f_u).*

Proof. Let $\psi = \prod_v \psi_v$ be a non-trivial additive character of $\mathbb{A}$ *mod* F. Denote by $L(s,\pi_v)$ the L-function, and by $\epsilon(s,\pi_v,\psi_v)$ the ϵ-factor, attached to π_v and ψ_v for every place v of F, in [GJ], Thm. 3.3. Consider $\pi = \pi_u \otimes \pi_0^u$ which occurs in (**4**) with $m(\pi) \neq 0$. Since π is automorphic, $\epsilon(s,\pi) = \prod_v \epsilon(s,\pi_v,\psi_v)$ is independent of ψ (see [GJ], p. 149), and $L(s,\pi) = \prod_v L(s,\pi_v)$ satisfies the functional equation $L(s,\pi) = \epsilon(s,\pi)L(1-s,\check{\pi})$; $\check{\pi}$ signifies the contragredient of π. Consequently, if π_u and π_{0u} contribute to (**4**) (namely $m(\pi_u \otimes \pi_0^u) \neq 0$, $m(\pi_{0u} \otimes \pi_0^u) \neq 0$), we have

$$\frac{L(1-s,\check{\pi}_u)\epsilon(s,\pi_u,\psi_u)}{L(s,\pi_u)} = \frac{L(1-s,\check{\pi}_{0u})\epsilon(s,\pi_{0u},\psi_u)}{L(s,\pi_{0u})}. \tag{5}$$

Denote by K_u the multiplicative group of a maximal order in the division algebra D_u underlying G_u. K_u is open in G_u, and G_u/Z_u is compact. Hence there are only finitely many irreducible G_u-modules π_u with a trivial central character which are unramified (trivial on K_u). If π_u is not trivial on K_u, then $L(s,\pi_u) = 1 = L(s,\check{\pi}_u)$ by [GJ], Prop. 4.4, identically in s. In this case (**5**) implies that $\epsilon(s,\pi_u,\psi_u)$ is independent of π_u (as long as $m(\pi_u \otimes \pi_0^u) \neq 0$). Denote by $c(\pi_u)$ the positive integer ("conductor") such that π_u is trivial on $1 + \boldsymbol{\pi}_u^{c(\pi_u)+1} R_u$ but not on $1 + \boldsymbol{\pi}_u^{c(\pi_u)} R_u$, where $\boldsymbol{\pi}_u$ is the local uniformizer in the ring R_u of integers in F_u. Choose ψ_u to be trivial on R_u, but not on $\boldsymbol{\pi}_u^{-1} R_u$. It is well-known (see, e.g., [BF], Thm 3.2.11, p. 39; this reference was pointed out to me by Waldspurger), that there exists a constant α such that $\epsilon(s,\pi_u,\psi_u) = \alpha q_u^{-c(\pi_u)s}$; q_u is the cardinality of the residue field $R_u/(\boldsymbol{\pi}_u)$. Consequently $c = c(\pi_u)$ is independent of π_u. Since $G_u/Z_u(1+\boldsymbol{\pi}_u^c R_u)$ is finite, there are only finitely many irreducible G_u-modules π_u with a trivial central character and a fixed conductor c. The proposition follows.

Now that we know that the sum in (**4**) ranges over a finite set depending only on π'_{0u}, a simple application of the Weyl integration formula implies

PROPOSITION 4. *For every regular γ in G_u and γ' in G'_u with $p_\gamma = p_{\gamma'}$ we have*

$$(-1)^{n-1}\ \chi_{\pi'_{0u}}(\gamma') = \sum_{\pi_u} m(\pi_u \otimes \pi_0^u)\ \chi_{\pi_u}(\gamma); \tag{6}$$

the sum is the same as in (**4**).

An immediate application of the orthonormality relations of characters of square-integrable representations, due to [K1], Theorem K, implies

PROPOSITION 5. *The sum in* (**6**) *consists of a single entry π_{0u} with $m(\pi_{0u} \otimes \pi_0^u) \neq 0$; moreover, $m(\pi_{0u} \otimes \pi_0^u) = 1$.*

This completes the proof of one half of the Theorem, asserting that for each square-integrable π'_u there exists a corresponding π_u. To prove the opposite direction one starts with a G_u-module π_u^0 and constructs a cuspidal $G(\mathbb{A})$-module π_0 whose component at u is π_u^0, at u'' it is the supercuspidal $\pi_{u''}^0$, and it is $\mathbf{1}_{u'}$ at u'. Then (**4**) is obtained and the proof proceeds as above.

Finally we use the Theorem to transfer orbital integrals. Since the following discussion is purely local the index u is omitted. Recall that for a regular γ in G', the centralizer G'_γ

is a torus, and we put

$$\Phi(\gamma, f') = \int_{G'/G'_\gamma} f'(g\gamma g^{-1}) dg.$$

Following [K1] we say that f' is *discrete* if $\Phi(\gamma, f') = 0$ for every regular non-elliptic γ in G'. The space of discrete f' is denoted by $A(G')$. The Theorem has the following

COROLLARY. *For every f on G there is f' in $A(G')$, and for every f' in $A(G')$ there is f on G, with $\Phi(\gamma, f) = \Phi(\gamma', f')$ for all regular γ in G and γ' in G' with $p_\gamma = p_{\gamma'}$.*

The proof consists of two parts.

LEMMA 1. *For every f there is f' in $A(G')$, and for every f' in $A(G')$ there is f, such that $(-1)^{n-1} tr\ \pi(f) = tr\ \pi'(f')$ for all π, π' corresponding as in the Theorem.*

Proof. Given f, define a form Φ on the free abelian group $R(G')$ generated by the equivalence classes of irreducible tempered G'-modules π' by $\Phi(\pi') = (-1)^{n-1}\ tr\ \pi(f)$ if π' is square-integrable and it corresponds to π, and by $\Phi(\pi') = 0$ if π' is irreducible, tempered but not square-integrable. It is clear that Φ is a good form in the terminology of [BDK], hence a trace form by the Theorem of [BDK]. Namely there exists f' on G with $\Phi(\pi') = tr\ \pi'(f')$ on $R(G')$. Since $tr\ \pi'(f') = 0$ for every π' in $R_I(G')$ (in the notations of [K1]), we have that f' lies in $A(G')$. The proof of the opposite implication (given f' in $A(G')$, there is f on G) is analogous.

LEMMA 2. *If f' in $A(G')$ and f on G satisfy $(-1)^{n-1} tr\ \pi(f) = tr\ \pi'(f')$ for all π, π' corresponding by the Theorem, then $\Phi(\gamma, f) = \Phi(\gamma', f')$ for all regular γ, γ' with $p_\gamma = p_{\gamma'}$.*

Proof. The Weyl integration formula implies that

$$\sum_{\{T\}} [W(T)]^{-1} \Delta(t)^2 \int_{T/Z} [\Phi(t, f) - \Phi(t', f')]\ \chi_\pi(t)\ dt = 0$$

for every G-module π. Here the sum ranges over a set of representatives for the conjugacy classes of tori T in G, $[W(T)]$ denotes the cardinality of the Weyl group of T, and Δ is a Jacobian. Since G/Z is compact, the characters χ_π form an orthonormal basis with respect to the inner product

$$< \chi, \chi' >= \sum_{\{T\}} [W(T)]^{-1} \Delta(t)^2 \int_{T/Z} \chi(t)\ \overline{\chi}'(t)\ dt.$$

The lemma follows, and so does the corollary.

CONCLUDING REMARK. Our Theorem and Corollary are the initial, special case of the correspondence of representations of $GL(n)$ and its inner forms; see [F1], III, §0, Local Theorem, for the general statement for the multiplicative group of any simple, not only division, algebra. Our local Theorem has a global variant, relating cuspidal representations on $GL(n, \mathbb{A})$ and $G(\mathbb{A})$, for any inner form G of $GL(n)$; see [F1], III, §0, Global Theorem, in the context of π' with two supercuspidal components, and [FK] in the context of π' with a single supercuspidal component.

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