SECOND MIDTERM INFORMATION

86
82
81
73
69
68
67
64
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61
49
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42
38
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23
7

1. $(\forall n,d \in Z)(d > 0 \rightarrow (\exists! q,r \in Z)(n = dq+r \wedge 0 \leq r < d))$.

2. Every integer > 1 can be written uniquely as a product of primes $p_1 \bullet p_2 \bullet\bullet\bullet p_k$, where $p_1 \leq ... \leq p_k$, and $k \geq 1$.

$150 = 2^1 \bullet 3^1 \bullet 5^2$.

3.
THEOREM. $(\forall n,m,r \in Z)(n \text{ odd} \wedge m \text{ odd} \wedge r \text{ odd} \rightarrow n+m+r \text{ odd})$.

Proof: Let n,m,r ∈ Z. Assume n odd ∧ m odd ∧ r odd. Want n+m+r odd. Have n odd, m odd, r odd. By definition of odd, let n = 2x+1, m = 2y+1, r = 2z+1, where x,y,z ∈ Z. Then n+m+r = 2x+1+2y+1+2x+1 = 2(x+y+z)+3 = 2(x+y+z+1)+1, which is odd by definition. QED

4.
THEOREM. $(\forall x \in \Re\backslash Q)(\forall y \in Q)(x+y^2 \in \Re\backslash Q)$.

Proof: Let $x \in \Re\backslash Q$. Let $y \in Q$. Want $x+y^2 \in \Re\backslash Q$. Have $x+y^2 \in \Re$. Want $x+y^2 \notin Q$. Assume x+y2 ∈ Q. Want ⊥ (symbol for contradiction). Since y ∈ Q, we have $y^2 \in Q$, and so x ∈ Q. This contradicts $x \in \Re\backslash Q$. QED

5.
THEOREM. $(\forall n \in Z)(2-n^2 \text{ is not divisible by } 3)$.

Proof: Let n ∈ Z. By the Quotient Remainder Theorem with d = 3 applied to n, write n = 3q+r, q ∈ Z, 0 ≤ r < 3. There are three cases according to the value of r.

case 1. n = 3q. Then $2-n^2 = 2-9q^2$. Hence $(2-n^2)/3 = 2/3 - 3q^2$, which is not an integer.

case 2. n = 3q+1. Then 2-n2 = $2-(3q+1)^2$ = 2-(9q2+6q+1) = -9q2-6q+1. Hence $(2-n^2)/3 = -3q^2-2q + 1/3$, which is not an integer.

case 3. n = 3q+2. Then $2-n^2 = 2-(3q+2)^2 = 2-(9q^2+12q+4)$ = -9q2-12q-2. Hence $(2-n^2)/3 = -3q^2-4q - 2/3$, which is not an integer.

QED

6. n ≥ 5. For 0 ≤ n < 5, we have, respectively,
11 < 0
13 < 1
15 < 4
17 < 9
19 < 16

THEOREM. $(\forall n \in Z)(n \geq 5 \rightarrow 2n+11 < n^2)$.

Proof: By ordinary induction on n ≥ 5.

Basis. n = 5. $21 < 5^2$. True.
Induction Hypothesis. Assume $2n+11 < n^2$, n ≥ 5.
Induction Conclusion. Want $2(n+1)+11 < (n+1)^2$.

Want $2n+13 < n^2+2n+1$. Want $2n+12 < n^2+2n$. Have $2n+12 < n^2+1$. Want $n^2+1 < n^2+2n$. Want 1 < 2n. Obvious, since n ≥ 5. QED

7. THEOREM. $(\forall n \geq 1)(a_n = 2^{n-1})$.

Proof: By strong induction on n ≥ 1.

Basis. n = 1,2. $a_1 = 1 = 2^{1-1}$. $a_2 = 2 = 2^{2-1}$.
Strong Induction Hypothesis. Have $(\forall i)(1 \leq i < n \rightarrow a_i = 2^{i-1})$, n > 2.
Strong Induction Conclusion. Want $a_n = 2^{n-1}$.

Since n ≥ 3, we have $a_n = a_{n-1} + 2a_{n-2} = 2^{n-2} + 2(2^{n-3}) = 2^{n-2} + 2^{n-2} = 2^{n-1}$. QED