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First Midterm Review Sheet
Midterm: January 25, 2008

Covers 1.1,1.2,1.3,2.1,2.2,2.3,2.4.

Know definitions listed below. You must ***feel*** definitions and proofs, rather than memorize them.

1. Conjunction, disjunction, implication, biconditional, negation.
2. Logical equivalence, tautologies, contradictions.
3. Converse, inverse, contrapositive.
4. Necessary, sufficient.
5. Modus ponens, modus tollens.
6. Argument, premises, valid argument, invalid argument, critical row.

7. Predicate, domain, truth set.
8. Universal quantifier, existential quantifier.
9. Universal statement, existential statement, example, counterexample.
10. Universal conditional statement, and its contrapositive, converse, inverse.
11. Necessary, sufficient, with quantifiers present.
12. Universal instantiation, universal modus ponens, universal modus tollens.
13. **N**, **Z**, **Q**, **R**. See front cover.

Know how to

1. Make a truth table for a statement form. Use it to determine logical equivalence, tautologies, contradictions.
2. Use the precedence table on p. 24 to properly work with statement forms which conserve on parentheses.
3. Use truth tables to determine the validity/invalidity of arguments.
4. Use rules on page 40 in order to give proofs as in p. 43, #44.
5. Translate from English into logic with quantifiers.
6. Read logic statements with quantifiers. Write negations of statements with/without quantifiers.
7. Determine truth/falsity and give reasons, as on p. 110, #40, and as we discussed for 3 days.
8. Use diagrams to determine the validity/invalidity of simple arguments with quantifiers as in 2.4.

METHODS OF PROOF THUS FAR:

1. Suppose you Want (to prove) a universal statement. Let the universal quantifier(s) be given and Want (to prove) the inside.
2. Suppose you Want (to prove) an existential statement. Give an example for the existential quantifier(s), and Want (to prove) the inside. The example may involve variables.
3. Suppose you Want (to prove) an implication. Assume the left side and Want (to prove) the right side.
4. Suppose you Want (to prove) a conjunction. Want (to prove) each conjunct.
5. Suppose you Want (to prove) a disjunction. Want (to prove) one of the disjuncts.
6. Suppose you Want (to prove) a negation. Assume the statement and Want (to prove) a contradiction.
7. Suppose you already Have a conjunction. To use it, Have (use) each conjunct.
8. Suppose you already Have $A \to B$ and already Have A. Then conclude (have) B.
9. Suppose you already have $(\forall x)(A(x))$. You can apply it by setting x to be t, and concluding A(t).

SOME HOMEWORK PROBLEMS

1.1. 2a. $p \to q, \neg q \therefore \neg p$. 7. $m \wedge \neg c$. 24. Logically equivalent. 40. (num_orders ≥ 50 or num_instock ≤ 300) and (50 > num_orders or num_orders ≥ 75 or num_instock ≤ 500).

1.2. 17. Let p = "Rob is ...", q = "Aaron play...", r = "Sam plays...". $p \wedge q \to r$. $\neg p \vee \neg q \vee r$. Yes, logically equivalent by truth tables. 27. Start with $p \to q$. Converse is $q \to p$. Inverse is $\neg p \to \neg q$. Converse is contrapositive of the inverse. So converse and inverse are logically equivalent. 33. If Sam will ... then he is ... . If Same is not ... then Sam will not ... .

1.3. 9. Not valid. Let p = T, q = F, r = T. 28. $p \to q, q, \therefore p$. Converse error. 44.

1. $p \to q$
2. $r \vee s$
3. $\neg s \to \neg t$
4. $\neg q \vee s$
5. $\neg s$
6. $\neg p \wedge r \to u$
7. $w \vee t$

$\therefore u \wedge w$

8. ¬q 4,5
9. ¬p 1,8
10. ¬t 3,5
11. w 7,10
12. r 2,5
13. ¬p ∧ r 9,12
14. u 6,13
15. u ∧ w 11,14

28. p → q
∴ q → p.
converse error.

32. p → r
q → r
∴ p ∨ q → r.
Valid. By truth tables.

2.1. 1a. false. b. true. c. false. d. true. e. false. f. true. 7a. {-6, -3, -2, -1, 1, 2, 3, 6}. b. {1,2,3,6}. c. [1,2] ∪ [-2,-1]. d. {1,2,-1,-2}. 12. Let x = y = 1. Left is sqrt(2), right is 2. 26. a. true. b. true. c. true. d. true. Let x = 1/2. 30. I prefer to put (∀x) in front of each statement. a. true. b. true. c. false. Let x = -3. d. true.

2.2. 4. a. There is a pot without a lid. b. There is a bird that can't fly. c. There is a pig that can't fly. d. Every dog has no spots. 12. There is an irrational number and a rational number whose product is irrational. (∃d ∈ Z)(6/d ∈ Z ∧ d ≠ 3).

2.3. 6. true. 20. a. true, true. b. true, false. 40.
a. false. Counterexample: x = 1. Want ¬(∃y ∈ Z^+)(1 = y+1). Want (∀y ∈ Z^+)(¬1 = y+1). Let y ∈ Z^+. Want ¬1 = y+1. Assume 1 = y+1. Want contradiction. Have y = 0. Contradiction.
b. true. Let x ∈ Z. Want (∃y ∈ Z)(x = y+1). Example: y = x-1. Want x = (x-1)+1. Algebra.
c. false. Want ¬(∃x ∈ ℜ)(∀y ∈ ℜ)(x = y+1). Want (∀x ∈ ℜ)(∃y ∈ ℜ)(¬x = y+1). Let x ∈ ℜ. Want (∃y ∈ ℜ)(¬x = y+1). Example: y = x. Want ¬x = x+1. Algebra.
d. true. Let x ∈ $ℜ^+$. Want (∃y ∈ $ℜ^+$)(xy = 1). Example: y = 1/x. Want x(1/x) = 1. Algebra.
e. false. Counterexample: x = 0. Want ¬(∃y ∈ ℜ)(0y = 1). Want (∀y ∈ ℜ)(¬0y = 1). Let y ∈ ℜ. Want ¬0y - 1. Algebra.

f. false. Counterexample: $x = 1$, $y = 1$. Want $\neg(\exists z \in Z^+)(z = 1-1)$. Want $(\forall z \in Z^+)(\neg z = 0)$. Let $z \in Z^+$. Want $\neg z = 0$. Algebra.
g. true. Let $x,y \in Z$. Want $(\exists z \in Z)(z = x-y)$. Example: $z = x-y$. Want $x-y = x-y$. Algebra.
h. true. Example: $u = 1/2$. Want $(\forall v \in \Re^+)(v/2 < v)$. Let $v \in \Re^+$. Want $v/2 < v$. Want $v < 2v$. Algebra.
i. true. Let $v \in \Re^+$. Want $(\exists u \in \Re^+)(uv < v)$. Example: $u = 1/2$. Want $v/2 < v$. Algebra.

2.4. 6. computer program is not correct. 12. Invalid. Draw a circle for honest, surrounded by a circle for pay. Put a dot for Darth in pay but not in honest. 14. Invalid. Draw a circle for error, surrounded by a circle for “not correct”. Put a dot for ‘this’ in “not correct” but not in error. 20. a. Draw a circle for dogs, surrounded by a circle for carnivorous. Put a dot for Aaron inside carnivorous and outside dog. b. This shows that this inference is not valid. 24. Valid. Draw two disjoint circles, one for vegetarians, one for eat. Draw a third circle for vegans inside the vegetarian circle. See that the vegans circle is disjoint from the eat circle.