

366  
First Midterm  
Information  
1/31/08

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1. Contradiction.

2. A statement form is a tautology if and only if it is true under all truth assignments to the letters in the statement form.

A statement form is a contradiction if and only if it is false under all truth assignments to the letters in the statement form.

Two statement forms are logically equivalent if and only if they have the same truth value under all truth assignments to the letters in the two statements forms.

3. Valid. There is one critical row. It has  $p = F$  and  $q = T$ .

4. a)  $(\forall n \in \mathbb{Z}) (\sim n < n)$   
 b)  $(\forall n \in \mathbb{Z}) (n > 0 \rightarrow (\exists a, b, c, d \in \mathbb{Z}) (n = a^2 + b^2 + c^2 + d^2))$   
 c)  $(\exists x \in \mathbb{Q}) (\forall n, m \in \mathbb{Z}) (x \neq n+m)$
5. a) True. Let  $x \in \mathbb{Z}$ . Want  $(\exists y \in \mathbb{N}) (y > x^2)$ . Example:  $y = x^2 + 1$ . Want  $x^2 + 1 > x^2$ . Algebra.
- b) False. Counterexample:  $x = 1$ . Want  $\sim (\exists y \in \mathbb{Z}) (2y = x)$ .  
 Want  $(\forall y \in \mathbb{Z}) (\sim 2y = 1)$ . Let  $y \in \mathbb{Z}$ . Want  $\sim 2y = 1$ . Assume  $2y = 1$ . Want contradiction. Have  $y = 1/2$ . Contradiction. QED
- c) True. Example:  $x = -1$ . Want  $(\forall y \in \mathbb{Q}) (-1 < y^2)$ . Let  $y \in \mathbb{Q}$ . Want  $-1 < y^2$ . Algebra.
- d) False. Want  $\sim (\exists x \in \mathbb{Z}) (\forall y \in \mathbb{N}) (x > y)$ . Want  $(\forall x \in \mathbb{Z}) \sim (\forall y \in \mathbb{N}) (x > y)$ . Want  $(\forall x \in \mathbb{Z}) (\exists y \in \mathbb{N}) (\sim x > y)$ . Want  $(\forall x \in \mathbb{Z}) (\exists y \in \mathbb{N}) (x \leq y)$ . Let  $x \in \mathbb{Z}$ . Want  $(\exists y \in \mathbb{N}) (x \leq y)$ .  
 Example:  $y = x$ . Want  $x \leq x$ . Algebra.
- e) True. Example:  $x = 0, y = 0$ .
6. Valid. Circle for trivial inside circle for easy. Circle for worthwhile entirely to the right (or left) of circle for easy. Point for Logic inside circle for worthwhile. Note that Point for Logic is outside the circle for trivial.
- 7.
5.  $q \wedge r$  2,4  
 6.  $q$  5  
 7.  $r$  5  
 8.  $s$  1,7  
 9.  $q \wedge s$  6,8  
 10.  $u$  3,9