Isomorphism
between
Trains of Laser Pulses
and
Arrays of Laser Illuminated Slits

To: Fellow Fourier Transform Afficionados,

At our last Laser Lunch we were trying to establish the isomorphism between

1. a sequence of identically structured pulses in time with its concomitant Fourier spectrum and
2. a multi-slit radiation source with its concomitant far field interference-diffraction pattern (amplitude, NOT intensity), which by the Fraunhofer-Kirchhof theorem is simply the Fourier amplitude of the source amplitude.

Contrary to my initial claim, the validity of this isomorphism holds always if the multi-slit screen in (2.) gets illuminated by plane wave fronts which strike all the slits at once. In other words, the laser in back of the multislit screen is always directed perpendicularly to that screen.

In order to take advantage of the isomorphism I propose that we smoke out the interference-diffraction pattern for a number of differently structured slit sources corresponding to different experiments.

I. Unstructured slits

a) A single unstructured slit of width L (on page 18-10 of the hand written handout, or of width ϵ on page 18-5 of the same handout)

b) Two unstructured slits whose separation s is large compared to the slit width L: $1 << s/L$.

c) K unstructured slits, equally spaced, whose nearest neighbor separation s is large compared to the slit width L: $1 << s/L << K$.

Question: Is the diffference between the diffraction amplitude patterns (Fourier amplitudes) for $1 << s/L << K$ and $1 << K << s/L$ important?

II. Identically structured slits, each one containing a periodic transmission grating having ℓ maxima so that

$$\begin{aligned} transmissivity &= A + B\cos\left(\frac{2\pi\ell x}{L}\right) \qquad \text{(see page 18-5 of that handout)} \\ &= A + \frac{B}{2}\exp\left(i\frac{2\pi\ell x}{L}\right) + \frac{B}{2}\exp\left(-i\frac{2\pi\ell x}{L}\right) \end{aligned}$$

a) A single such structured slit.

b) Two such structured slits whose separation s is large compared to the slit width L: $1 << s/L$ and ℓ is any integer.

c) K such structured slits, equally spaced, whose nearest separation is large compared to the slit width L: $1 << s/L << K$, $1 << K << s/L$.

I believe that it is safe to say that, once we understand the Young's multi-slit interference-diffraction pattern, we understand the Fourier transform of the corresponding train of laser pulses in the time domain.

Sincerely,

Ulrich Gerlach

P.S. The nature of the isomrphism between a train of laser pulses and a Young's multi-slit interference of diffraction patterns is is highlighted in parts 1 and 4 of the hand written handout, copies of which I will leave in SM 0064. The theoretical principle underlying "sinc"-shaped pulses (a.k.a. wave packets) is found in another handout, which is a continuation of the handout from 3 or 4 weeks ago.

P.P.S. We still must come back to Richard Hamming's "You and Your Research," and report on which of his idea(s) is most appealing to you.

P.P.P.S. We must think about how to measure the phase of the optical carrier of a train of laser pulses.