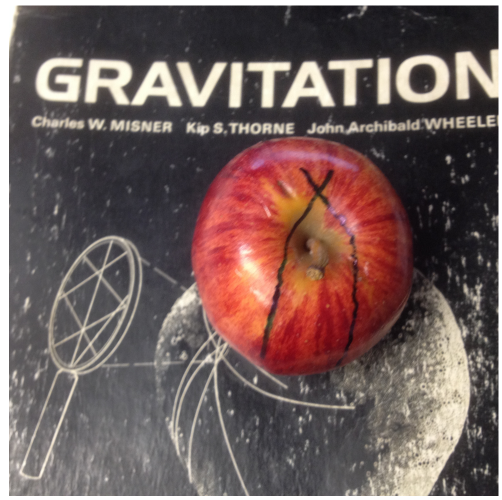
COURSE ANNOUNCEMENT and SYLLABUS
Mathematics 5756 (Autumn 2022)
Mathematics 5757 (Spring 2023)



Course Name: Modern Mathematical Methods in Relativity Theory I, II (a.k.a. "Applied Differential Geometry")
Class Number: 35486 (undergrad), 35487 (graduate)
Time and Place: Autumn 2022: MWF 12:40pm; JR 139.
Office Hours: Usually right after class
Credits: 3 per semester
Prerequisites:
(i) Multivariable differential calculus and linear algebra (e.g. Math 2568 and/or 5101, 5522H).
(ii) By the 4th week one should have reviewed the concept of *"the differential of a function"* so as to have grasped the relation to the *"directional derivative of that function"*.
(iii) A physics course (e.g. Physics 1250 or higher).
(iv) No prior knowledge of tensor calculus is assumed. However, we do assume a mature attitude towards mathematics and physics.
Audience: Mature undergraduate and graduate
Textbooks:
(i) *Gravitation* by C. W. Misner, K. S. Thorne, and J. A. Wheeler (see above).
(ii) Selections from *Spacetime Physics*, 2nd edition, by E. Taylor and J.A. Wheeler
(iii) Selections from *Mathematical Methods of Classical Mechanics* by V. I. Arnold.
(iv) Selections from *Lecture Notes on Elementary Topology and Geometry* by I. M. Singer.

Purpose, Goal and Benefits:

1. Develop *from the bottom up* the fundamental mathematical concepts and methods responsible for the successes in 20^{th} century physics, mathematics, and theoretical engineering.
   Thus Math 5756 concretizes these developments in terms of
   (a) Special Relativity as the cognitive bridge to 20^{th} century geometry,
   (b) multilinear algebra as a source of geometrical structures,
   (c) linear algebra's marriage to multi-variable calculus,

(d) differential geometry as a three level hierarchy characterized by its

- Differential structure,
- Parallel transport structure (a.k.a. covariant derivative)
- Metric structure.

(e) the exterior calculus

(f) Cartan's two structural equations and their application to the Cartan-Misner calculus.

2. From the mathematical perspective these developments serve as a prelude to global mathematics.

3. From the philosophic perspective these developments (and those in Math 5757) serve to illustrate that "Mathematics Is About the World".

Agenda:

a) Assimilate the mathematical chapters of our primary text ("*Gravitation*" by MTW, see references above), thus to develop an appreciation and the modern machinery for the mathematical framework of the laws of physics from the spacetime perspective. The development will focus on

(1) the underlying differential geometric framework of spacetime, and

(2) the formulation (arising from classical mechanics, fluid dynamics, and wave mechanics) of its properties.

b) Show *why* and how mathematics is the language of physical science, in particular of those aspects of physics dealing with processes of extreme violence (relativistic hydrodynamics, relativistic laser-matter interaction, high energy density physics, gravitational collapse in flat or curved spacetimes).

Exams:

All exams are in the form of *individual* open-book open-notes take-home exams.

Homework:

There will be about eight homework sets to be submitted in the form of Latex-based PDF's.

Website:

`https://people.math.osu.edu/gerlach.1/math5756`

DESCRIPTION

Math 5756 (Autumn):

- A rapid course in special relativity: spacetime geometry, event horizons and accelerated frames;
- tensors, metric geometry vs symplectic geometry;
- exterior calculus, Maxwell field equations;
- manifolds, Lie derivatives, and Hamiltonian dynamics in phase space;
- parallel transport, torsion, tensor calculus;
- curvature and Jacobi's equation of geodesic deviation;

- Cartan's two structural equations, metric induced properties, and Cartan-Misner curvature calculus.

Math 5757 (Spring):

- Geodesics: Hamilton-Jacobi theory, the principle of constructive interference;
- stress-energy tensor: hydrodynamics in curved spacetime;
- Einstein field equations: moment of rotation = momentum-energy;
- The conservation laws and the Bianchi identities mathematized in terms of the "Boundary of a Boundary is zero ($\partial\,\partial\,\Omega = 0$)" Principle.
- Solutions to the Einstein's field equations: stars, black holes, gravitational collapse, geometry and dynamics of the universe;
- vector harmonics, tensor harmonics, acoustic and gravitational waves in violent relativistic backgrounds.

I am glad to answer any questions.
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