

MATH 150 FINAL (W 2004) ANSWER KEYS

1. (a). Basic function: x^2 .

(1). $x^2 \rightarrow (x - 3)^2$: shift to the right by 3;

(2). $(x - 3)^2 \rightarrow 2(x - 3)^2$: vertical stretching by a factor of 2;

(3). $2(x - 3)^2 \rightarrow 2(x - 3)^2 - 2$: shift down by 2;

Note the order of the transformation! What happens if you switch step (2) and step (3)?

(b). Graph omitted. Some candidates of “good” points: $(3, -2)$, $(2, 0)$ and $(4, 0)$.

2. (a). x -intercepts: $x = 4$ (touches), $x = -2$ (touches). There is no symmetry.

(b). Turning points: $(-2, 0)$, $(1, 81)$, $(4, 0)$. Asymptotic behavior: x^4 .

3. (a). x -intercepts: none. y -intercepts: $y = -2$. Vertical asymptotes: $x = -1$, $x = 3$.

(b). Horizontal asymptotes: $y = 0$. Oblique asymptotes: none.

(c). Graph omitted. Make sure to plot all intercepts and asymptotes.

4. (a). $x = \frac{2 \ln 3 + \ln 5}{2 \ln 5 + \ln 3}$.

(b). $\frac{1}{3}[\ln(x - 2) + \ln(x + 2)] - [\ln 5 + 4 \ln x + 2 \ln(x + 1)]$.

(c). $x = 5$. Why do we discard the solution $x = -1$?

5. θ is in QII.

$$\sin \theta = \frac{\sqrt{13}}{7},$$

$$\cos \theta = \frac{-6}{7},$$

$$\tan \theta = \frac{\sqrt{13}}{-6},$$

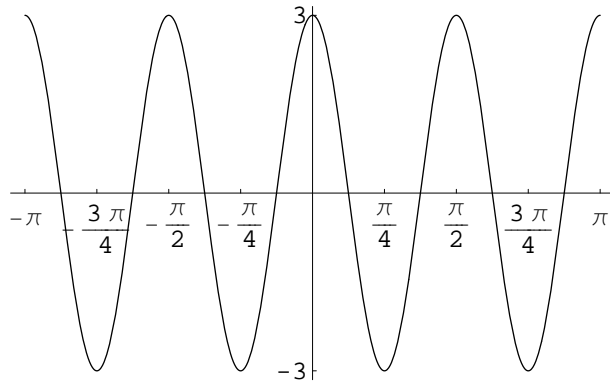
$$\csc \theta = \frac{7}{\sqrt{13}},$$

$$\sec \theta = \frac{7}{-6},$$

$$\cot \theta = \frac{-6}{\sqrt{13}}.$$

6. (a). amplitude = 3, period = $\frac{\pi}{2}$, phase shift = $-\frac{\pi}{4}$.

(b). 5 key points: $(-\frac{\pi}{4}, -3)$, $(-\frac{\pi}{8}, 0)$, $(0, 3)$, $(\frac{\pi}{8}, 0)$, $(\frac{\pi}{4}, -3)$.

FIGURE 1. $-3 \cos(4x + \pi)$.

$$7. \sin^{-1} \left[\cos \left(\frac{7\pi}{6} \right) \right] = \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) = -\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) = -\frac{\pi}{3}.$$

$$8. \frac{2 - \frac{1}{\sqrt{3}}}{1 + 2 \cdot \frac{1}{\sqrt{3}}} = \frac{2\sqrt{3} - 1}{\sqrt{3} + 2}.$$

9.

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos \theta (1 + \cos \theta)}{1 + \cos \theta} + \frac{\sin^2 \theta}{1 + \cos \theta} && \text{(common the denominators)} \\ &= \frac{\cos \theta + \cos^2 \theta}{1 + \cos \theta} + \frac{\sin^2 \theta}{1 + \cos \theta} \\ &= \frac{\cos \theta + \cos^2 \theta + \sin^2 \theta}{1 + \cos \theta} \\ &= \frac{\cos \theta + 1}{1 + \cos \theta} && (\sin^2 \theta + \cos^2 \theta = 1) \\ &= 1 = \text{R.H.S.} \end{aligned}$$

$$10. \sin \left(\frac{11\pi}{8} \right) = \sin \left(\frac{1}{2} \cdot \frac{11\pi}{4} \right) = -\sqrt{\frac{1 - \cos \frac{11\pi}{4}}{2}} = -\sqrt{\frac{1 + \frac{1}{\sqrt{2}}}{2}}.$$

$$11. \theta = \pi + 6k\pi, \theta = -\pi + 6k\pi.$$

$$12. \text{(a). } x^2 + y^2 = 2x.$$

$$\text{(b). } (\sqrt{2})^8 (\cos 120^\circ + i \cdot \sin 120^\circ) = -8 + i \cdot 8\sqrt{3}.$$