

## MATH 150 MIDTERM 1 (W 2005) ANSWER KEYS

1. (a).  $x$ -intercepts:  $x = 0$  (crosses),  $x = -2$  (touches). There is no symmetry.  
(b). Turning points:  $(-2, 0)$ ,  $(-0.809, 8.387)$ . Asymptotic behavior:  $-2x^5$ .
  
2. (a).  $x$ -intercepts: none.  $y$ -intercepts:  $y = -\frac{4}{3}$ . Vertical asymptotes:  $x = -1$ ,  $x = 3$ .  
(b). Horizontal asymptotes:  $y = 0$ . Oblique asymptotes: none.  
(c). Graph omitted. Make sure to plot all intercepts and asymptotes.
  
3.  $(-\infty, -2] \cup (-1, 3]$  or  $\{x : x \leq -2 \text{ or } -1 < x \leq 3\}$ . Why the end points  $-2$  and  $3$  are included while  $-1$  is not?
  
4. (a). Basic function:  $\log_2 x$ .  
(1).  $\log_2 x \rightarrow \log_2(x + 1)$ : shift to the left by 1;  
(2).  $\log_2(x + 1) \rightarrow -\log_2(x + 1)$ : reflection about the  $x$ -axis;  
(3).  $-\log_2(x + 1) \rightarrow 3 - \log_2(x + 1)$ : shift up by 3;  
Note the order of the transformation! What happens if you switch step (2) and step (3)?  
(b). Graph omitted. Some candidates of “good” points:  $(0, 3)$ ,  $(1, 2)$  and  $(3, 1)$ .
  
5. (a). Domain of  $f$ :  $\{x : x > -2\}$ . Range of  $f$ :  $\{y : y > 0\}$ .  $f$  is one-to-one because  $f$  is strictly decreasing (don't say because  $f$  passes the horizontal line test. This is not considered a valid explanation).  
(b).  $f^{-1}(x) = \frac{1}{x^2} - 2$ . Domain of  $f^{-1} =$  Range of  $f$ :  $\{x : x > 0\}$ . Range of  $f^{-1} =$  Domain of  $f$ :  $\{y : y > -2\}$ .
  
6. (a).  $x = \frac{\ln 3}{2 \ln 5 + \ln 3}$ .  
(b).  $x = 5$ . Why do we discard another solution  $x = -2$ ?