

MATH 851 HOMEWORK I (DUE NOV 23)

1. Cao Ch.1: 2, 5.
2. Let (M^n, g) be a Riemannian manifold and $p \in M$. Locally the exponential map $\exp_p : T_p M \rightarrow M$ is a diffeomorphism and thus defines a local coordinate system in a neighborhood of p :

$$x = (x_1, \dots, x_n) \mapsto \exp_p(x), \quad x \in T_p M = \mathbb{R}^n.$$

This is called the *normal coordinates* at p .

Prove that $g_{ij}(p) = \delta_{ij}$ and $\Gamma_{ij}^k(p) = 0$. (Note: This holds only at p .)

3. DoCarmo Ch.3: 7, 8, 9.
4. Cao Ch.2: 1, 2, 3, 4.
5. DoCarmo Ch.4: 5, 6.
6. Prove the second comparison theorem (Cao Theorem 2.6).