

MATH 851-852: DIFFERENTIAL GEOMETRY

- Riemannian manifolds
 - (1) Differentiable manifolds
 - (2) Tangent and cotangent bundles
 - (3) Riemannian metrics
 - (4) Volume elements and integrals
 - (5) Gradients
- Connections
 - (1) Affine connections
 - (2) Levi-Civita connection
 - (3) Parallel transport
 - (4) Hessian, Laplacian
- Geodesics
 - (1) Existence and uniqueness
 - (2) First variation formula
 - (3) Exponential map and Gauss lemma
 - (4) Completeness and Hopf-Rinow theorem
- Curvature
 - (1) Riemannian curvature tensor
 - (2) Sectional curvatures
 - (3) Ricci and scalar curvatures
 - (4) Tensors
- Jacobi fields
 - (1) The Jacobi equation
 - (2) Conjugate points
 - (3) Second variation formula
- Comparison theorems
 - (1) Riccati equation
 - (2) Gromov lemma
 - (3) Rauch comparison theorem
 - (4) Gromov-Bishop comparison theorem

- (5) Meyer theorem
- (6) Cheng theorem
- (7) Calabi-Yau volume estimate
- (8) Cartan-Hadamard theorem
- Injectivity radius
 - (1) The cut locus
 - (2) Injectivity radius
 - (3) Cheeger estimate
- Submanifolds
 - (1) Induced metric
 - (2) Second fundamental form
 - (3) The normal bundle connection
 - (4) Gauss-Codazzi equations
 - (5) Mean curvature
 - (6) Variation formulas
 - (7) Minimal submanifolds

Additional topics:

Complex geometry: Complex manifolds, Kähler geometry, Calabi-Yau theorem.

Bochner techniques.

Triangle comparison theorems.

Space forms.