Truth Tables, Basic Equivalencies, Tautologies and Contradictions

Truth tables are not a primary focus in Math 345; however, it is important to know the truth tables of the logical connectives. It is also important to understand how a truth table can be used to determine the overall truth values of a given sentence. Since a sentence with n logical variables would require a truth table with 2^n rows, it is necessary for you to learn more refined methods of proof.

Here are the truth tables of the basic logical connectives. They are all given in one large table:

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T

A **tautology** is a compound sentence that is always true and a **contradiction** is a compound sentence that is always false.

The some of the basic equivalencies that you should know are the following.

De Morgan's Laws:

(a.) $\neg(P \wedge Q)$ is logically equivalent to $(\neg P) \vee (\neg Q)$

(b.) $\neg(P \vee Q)$ is logically equivalent to $(\neg P) \wedge (\neg Q)$

Distributive Laws:

(a.) $P \wedge (Q \vee R)$ is logically equivalent to $(P \wedge Q) \vee (P \wedge R)$

(b.) $P \vee (Q \wedge R)$ is logically equivalent to $(P \vee Q) \wedge (P \vee R)$

Conditionals:

(a.) $P \Rightarrow Q$ is logically equivalent to $\neg P \vee Q$

(b.) $\neg(P \Rightarrow Q)$ is logically equivalent to $P \wedge \neg Q$

Bionditionals:

(a.) $P \Leftrightarrow Q$ is logically equivalent to $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

(b.) $P \Leftrightarrow Q$ is a tautology is the same as saying P is logically equivalent to Q