1.1 Equally Likely Outcomes

1. Introduction

We start with the concept of an experiment or **trial**. When this is performed there are several **possible outcomes**. We denote the set of all possible outcomes by Ω. An **event** is some subset A of all of the possible outcomes, so A is a subset of Ω. A is often called the set of **favorable outcomes**. Finally let $\sharp(A)$ or $|A|$ be the number of outcomes in A.

The notion of an **equally likely outcome** is that any outcome in Ω has an equal chance of occurring. In this case we define the probability $P(A)$ of the event A occurring to be:

$$P(A) = \frac{\sharp(A)}{\sharp(\Omega)}$$

It should be noted that determining probabilities is usually a question of counting outcomes.

2. Examples

Example 1: A 20 sided die is rolled. What is the probability that a 1 occurs?

In this case $\Omega = \{1, 2, \dots, 20\}$, these are all the possible outcomes. The event we are interested is when a 1 is rolled. Thus the set of favorable outcomes is $A = \{1\}$. Since $\sharp(A) = 1$ and $\sharp(\Omega) = 20$,

$$P(A) = \frac{1}{20}$$

Example 2: A 20 sided die is rolled. What is the probability that the result is an even number?

As above, $\Omega = \{1, 2, \dots, 20\}$. The event that we are interested in this time is that an even number is rolled, that is 2,4,6,8,10,12,14,16,18 or 20 is rolled. In this case, the set of favorable outcomes is $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$. Since $\sharp(A) = 10$ and $\sharp(\Omega) = 20$,

$$P(A) = \frac{10}{20} = \frac{1}{2}$$

Example 3: Two cards are dealt from a standard deck of 52 playing cards. What is the probability that both cards are Kings?

In this case, explicitly describing Ω is impractical. We only need to determine the size of Ω to determine the probability. If we consider the cards as dealt in order, then Ω consists of ordered pairs of cards. There are exactly 52 possible first cards and 51 possible second cards (one having already been dealt). Thus there are 52*51=2652 possible ordered pairs of cards. This means $\sharp(\Omega) = 2652$. Now the set of favorable outcomes A consists of those ordered pairs which have a King for both the first and second card. Since there are 4 Kings in a deck, there are 4*3=12 possible ordered pairs of two Kings. This means $\sharp(A) = 12$, which gives:

$$P(A) = \frac{12}{2652} = \frac{1}{221}$$