

Solution to 3.2.3

(a.)

Let X be the number of sixes appearing on 3 die rolls. Then

$$P(X = 0) = \binom{3}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$P(X = 1) = \binom{3}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2 = \frac{75}{216}$$

$$P(X = 2) = \binom{3}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^1 = \frac{15}{216}$$

$$P(X = 3) = \binom{3}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0 = \frac{1}{216}$$

So

$$E(X) = \sum_{k=0}^3 kP(X = k) = \frac{\mathbf{1}}{\mathbf{2}}$$

(b.) Let Y be the number of odd numbers that appear on 3 die rolls. Then

$$P(X = 0) = \binom{3}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(X = 1) = \binom{3}{1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

$$P(X = 2) = \binom{3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = \frac{3}{8}$$

$$P(X = 3) = \binom{3}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 = \frac{1}{8}$$

So

$$E(X) = \sum_{k=0}^3 kP(X = k) = \frac{12}{8} = \frac{\mathbf{3}}{\mathbf{2}}$$