Math 530

Autumn 2006

Solution to 3.3.17

Let X_i be equally the 25 random variables and let $S = X_1 + \cdots + X_{25}$. Then

$$E(S) = \sum_{i=1}^{25} E(X_i) = 25E(X_1) = 25\left(-1\frac{1}{4} + 0\frac{1}{4} + 1\frac{1}{2}\right) = \frac{25}{4} = 6.25$$

and

$$Var(S) = \sum_{i=1}^{25} Var(X_i) = 25Var(X_1) = 25E(X_1^2) - 25[E(X_1)]^2 = 25\left(1\frac{1}{4} + 0\frac{1}{4} + 1\frac{1}{2}\right) - \frac{25}{16} = \frac{275}{16} = 17.1875$$

so

$$SD(X) = 4.1458$$

(a.)

$$P(S < 0) = P(S \leq -1) \approx \Phi\left(\frac{-1 - 6.25}{4.1458}\right) = \Phi(-1.75) = 1 - \Phi(1.75) = 1 - 0.9599 = \mathbf{0.0401}$$

(c.)

$$P(S > 0) = 1 - P(S \leq 0) \approx 1 - \Phi\left(\frac{-6.25}{4.1458}\right) = 1 - \Phi(-1.51) = 1 - 1 + \Phi(1.51) = \mathbf{0.9345}$$

(b.)

$$P(S = 0) = 1 - P(S < 0) - P(S > 0) = 1 - 0.0401 - 0.9345 = \mathbf{0.0254}$$