Math 530

Autumn 2006

Solution to 3.3.22

Let X_i be the value of roll i. Let $S_n = X_1 + \cdots + X_n$ and $\overline{X}_n = \frac{X_1+\cdots+X_n}{n}$ is the average of n rolls. We have:

$$E(X_i) = \sum_{k=1}^{6} kP(X = k) = 3.5$$

and

$$Var(X_i) = \sum_{k=1}^{6} k^2 P(X = k) - (3.5)^2 = 2.9167$$

so

$$SD(X_i) = 1.7078$$

Moreover

$$E(S_n) = 3.5n \text{ and } SD(S_n) = 1.7078\sqrt{n}$$

(a.)

$$P\left(\frac{41}{12} \le \overline{X}_n \le \frac{43}{12}\right) = P\left(\frac{41n}{12} \le S_n \le \frac{43n}{12}\right) = P\left(S_n \le \frac{43n}{12}\right) - P\left(S_n < \frac{41n}{12}\right) = P(S_n \le 3.5833n) - P(S_n < 3.4167n)$$

so

$$P\left(\frac{41}{12} \le \overline{X}_n \le \frac{43}{12}\right) \approx \Phi\left(\frac{3.5833n - 3.5n}{1.7078\sqrt{n}}\right) - \Phi\left(\frac{3.4167n - 3.5n}{1.7078\sqrt{n}}\right) = \Phi(0.0488\sqrt{n}) - \Phi(-0.0488\sqrt{n}) = 2\Phi(0.0488\sqrt{n}) - 1$$

n=105: $2\Phi(0.0488\sqrt{n}) - 1 = 2\Phi(0.0488\sqrt{105}) - 1 = 2\Phi(0.5) - 1 = \mathbf{0.3830}$

n=420: $2\Phi(0.0488\sqrt{n}) - 1 = 2\Phi(0.0488\sqrt{420}) - 1 = 2\Phi(1) - 1 = \mathbf{0.6826}$

n=1680: $2\Phi(0.0488\sqrt{n}) - 1 = 2\Phi(0.0488\sqrt{1680}) - 1 = 2\Phi(2) - 1 = \mathbf{0.9544}$

n=6720: $2\Phi(0.0488\sqrt{n}) - 1 = 2\Phi(0.0488\sqrt{6720}) - 1 = 2\Phi(4) - 1 = \mathbf{0.9999}$