Math 530

Autumn 2006

Solution to 3.3.8

(a.)

$$\mathbf{N = I_{A_1} + I_{A_2} + I_{A_3}}$$

(b.)

$$E(N) = E(I_{A_1}) + E(I_{A_2}) + E(I_{A_3}) = P(A_1) + P(A_2) + P(A_3) = \frac{1}{5} + \frac{1}{4} + \frac{1}{3} = \mathbf{\frac{47}{60}}$$

(c.) Note that N is itself and indicator random variable in this case, $N = I_{A_1 \cup A_2 \cup A_3}$. Moreover $N^2 = N$. So

$$Var(N) = E(N^2) - [E(N)]^2 = P(A_1 \cup A_2 \cup A_3) - \left(\frac{47}{60}\right)^2 = \frac{47}{60} - \left(\frac{47}{60}\right)^2 = \frac{47}{60}\frac{13}{60} = \mathbf{0.1697}$$

(d.) A_i independent means I_{A_i} are independent. Note also that $(I_{A_i})^2 = I_{A_i}$. So

$$Var(N) = Var(I_{A_1}) + Var(I_{A_2}) + Var(I_{A_3}) = \sum_{i=1}^{3} E[(I_{A_i})^2] - [E(I_{A_i})]^2 = \frac{2}{9} + \frac{3}{16} + \frac{4}{25} = \mathbf{0.5697}$$

(e.)

$$E(N^2) = 1P(N = 1) + 4P(N = 2) + 9P(N = 3)$$

Since $A_1 \subset A_2 \subset A_3$, for one event to occur, A_3 must occur, but A_2 does not, which also implies A_1 also does not occur. So $P(N = 1) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$. Similarly $P(N = 2) = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$ and $P(N = 3) = \frac{1}{5}$. So

$$Var(N) = E(N^2) - [E(N)]^2 = \frac{1}{12} + 4\frac{1}{20} + 9\frac{1}{5} - \left(\frac{47}{60}\right)^2 = \mathbf{1.7697}$$