Math 530

Autumn 2006

Solution to 3.4.2

Note that $P(D = k)$ is the probability that the first $k - 1$ balls are all of the same color, and the kth ball is of a different color. Given that the first ball can be any color, we then choose $k - 2$ of the same color, then one of the other color. So

$$P(D = k) = \left(\frac{\mathbf{1}}{\mathbf{2}}\right)^{\mathbf{k-2}} \frac{\mathbf{1}}{\mathbf{2}}$$

Let $X = D - 1$ then $D = X + 1$ and

$$P(X = k) = P(D - 1 = k) = P(D = k + 1) = \left(\frac{1}{2}\right)^{k-1} \frac{1}{2}$$

So X is geometrically distributed with parameter $\frac{1}{2}$.

$$E(D) = E(X + 1) = E(X) + 1 = 2 + 1 = \mathbf{3}$$

(See the online notes for this and for $SD(D)$ below)

$$Var(D) = Var(X + 1) = Var(X) + Var(1) = 2 + 0$$

$$SD(D) = \sqrt{\mathbf{2}}$$