Math 254
Autumn 2003

Final Exam Review

Note: Make sure to look over both midterm exams as well as the reviews for the midterms.

(1.) Find the total mass and center of mass of the solid which is bounded on the sides by the cylinder $x^2 + y^2 = 4$, bounded above by the cone $z = \sqrt{x^2 + y^2}$, and below by the xy-plane where the mass/density function is $\delta(x, y, z) = z + 3$.

(2.) Use a suitable transformation to evaluate $\displaystyle\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$ where R is the region bounded by the lines $x + y = 2$, $x + y = 4$, $x = 0$ and $y = 0$.

(3.) Find the divergence and curl of the given of the vector field $\mathbf{F}$ where:

(a.) $\mathbf{F}(x, y) = \frac{x}{x^2+y^2}\mathbf{i} + \frac{y}{x^2+y^2}\mathbf{j}$.

(b.) $\mathbf{F}(x, y, z) = 4xy\mathbf{i} + (2x^2 + 2yz)\mathbf{j} + (3z^2 + y^2)\mathbf{k}$.

(4.) Show that $\mathbf{F}$ is conservative by finding a potential function f for $\mathbf{F}$ where:

(a.) $\mathbf{F}(x, y) = (y^3 + 3x^2y)\mathbf{i} + (x^3 + 3y^2x)\mathbf{j}$.

(b.) $\mathbf{F}(x, y, z) = 2xz\mathbf{i} + 2yz\mathbf{j} + (x^2 + y^2)\mathbf{k}$.

(5.) Compute $\displaystyle\int_C f\, ds$ where:

(a.) $f(x, y) = x^2 - y^2$ and C is parameterized by $\mathbf{r}(t) = 5\cos t\mathbf{i} + 5\sin t\mathbf{j}$ where $0 \le t \le 2\pi$.

(b.) $f(x, y) = x^3 + 2xy^2 + 2x$ and C is parameterized by $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j}$ where $0 \le t \le 1$.

(6.) Compute $\displaystyle\int_C \mathbf{F} \cdot \mathbf{dr}$ where:

(a.) $\mathbf{F}(x, y) = y^3\mathbf{i} - x^2\mathbf{j}$ and C is parameterized by $\mathbf{r}(t) = e^{-2t}\mathbf{i} + e^t\mathbf{j}$ where $0 \le t \le \ln 2$.

(b.) $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ and C is parameterized by $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$ where $1 \le t \le 3$.

(c.) $\mathbf{F}(x, y) = (x^2y + x)\mathbf{i} + (xy^2 + y)\mathbf{j}$ and C is the boundary of the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$ oriented counterclockwise.

(7.) Find the amount of work required to move a particle from $(-2, 4)$ to $(2, 4)$ along the curve $y = x^2$ through a force field given by $\mathbf{F}(x, y) = 2x^2y\mathbf{i} + 4y^2\mathbf{j}$.

(8.) Show that the given integral is path independent and calculate this integral for the given initial and terminal points of C where:

(a.) $\displaystyle\int_C (2x + 1)\, dx + (3y^2)\, dy + (\frac{1}{z})\, dz$, initial point: $(1, 2, 1)$, terminal point: $(3, 4, 1)$.

(b.) $\displaystyle\int_C (2x\sin y + e^{3z})\, dx + (x^2\cos y)\, dy + (3xe^{3z} + 5)\, dz$, initial point: $(1, 0, 0)$, terminal point: $(2, \pi/2, 1)$.